John E. Sjöquist.

John E. [illegible]

John E. Sjöquist

WORKS ON

DESCRIPTIVE GEOMETRY,

AND ITS APPLICATIONS TO

ENGINEERING, MECHANICAL AND OTHER INDUSTRIAL DRAWING.

By S. EDWARD WARREN, C.E.

I. ELEMENTARY WORKS.

PRIMARY GEOMETRY. An introduction to geometry as usually presented; and designed, *first*, to facilitate an earlier beginning of the subject, and, *second*, to lead to its graphical applications in manual and other elementary schools. With numerous practical examples and cuts. Large 12mo, cloth, 75c.

Industrial Science Series Comprising:

1. FREE-HAND GEOMETRICAL DRAWING, widely and variously useful in training the eye and hand in accurate sketching of plane and solid figures, lettering, etc. 12 folding plates, many cuts. Large 12mo, cloth, $1.00.

2. DRAFTING INSTRUMENTS AND OPERATIONS. A full description of drawing instruments and materials, with applications to useful examples; tile work, wall and arch faces, ovals, etc. 7 folding plates, many cuts. Large 12mo, cloth, $1.25.

3. ELEMENTARY PROJECTION DRAWING. Fully explaining, in six divisions, the principles and practice of elementary plan and elevation drawing of simple solids; constructive details; shadows; isometrical drawing; elements of machines; simple structures. 24 folding plates, numerous cuts. Large 12mo, cloth, $1.50.

This and No. 3 are especially adapted to scientific, preparatory, and manual-training industrial schools and classes, and to all mechanics for self-instruction.

4. ELEMENTARY PERSPECTIVE. With numerous practical examples, and every step fully explained. Revised Edition (1891). Numerous cuts. Large 12mo, cloth, $1.00.

5. PLANE PROBLEMS on the Point, Straight Line, and Circle. 225 problems. Many on Tangencies, and other useful or curious ones. 150 woodcuts, and plates. Large 12mo, cloth, $1.25.

II. HIGHER WORKS.

1. THE ELEMENTS OF DESCRIPTIVE GEOMETRY, SHADOWS AND PERSPECTIVE, with brief treatment of Trehedrals; Transversals; and Spherical, Axonometric, and Oblique Projections; and many *examples for practice*. 24 folding plates. 8vo, cloth, $3.50.

2. PROBLEMS, THEOREMS, AND EXAMPLES IN DESCRIPTIVE GEOMETRY. Entirely distinct from the last, with 115 problems, embracing many useful constructions; 52 theorems, including examples of the demonstration of geometrical properties by the method of projections; and many examples for practice. 24 folding plates. 8vo, cloth, $2.50.

3. GENERAL PROBLEMS IN SHADES AND SHADOWS, with practical examples, and including every variety of surface. 15 folding plates. 8vo, cloth, $3.00.

4. GENERAL PROBLEMS IN THE LINEAR PERSPECTIVE OF FORM, SHADOW, AND REFLECTION. A complete treatise on the principles and practice of perspective by various older and recent methods: in 98 problems, 24 theorems, and with 17 large plates. Detailed contents, and numbered and titled topics in the larger problems, facilitate study and class use. Revised edition, corrections, changes and additions. 17 folding plates. 8vo, cloth, $3.50.

5. ELEMENTS OF MACHINE CONSTRUCTION AND DRAWING. 73 practical examples drawn to scale and of great variety; besides 30 problems and 31 theorems relating to gearing, belting, valve-motions, screw-propellers, etc. 2 vols., 8vo, cloth, one of text, one of 34 folding plates. $7.50.

6. PROBLEMS IN STONE CUTTING. 20 problems, with examples for practice under them, arranged according to dominant surface (plane, developable, warped or double-curved) in each, and embracing every variety of structure; gateways, stairs, arches, domes, winding passages, etc. Elegantly printed at the Riverside Press. 10 folding plates. 8vo, cloth, $2.50.

INDUSTRIAL SCIENCE DRAWING.

ELEMENTARY PROJECTION DRAWING.

THEORY AND PRACTICE.

FOR PREPARATORY AND HIGHER SCIENTIFIC SCHOOLS; INDUSTRIAL AND NORMAL CLASSES; AND THE SELF-INSTRUCTION OF TEACHERS, INVENTORS, DRAFTSMEN, AND ARTISANS.

In Six Divisions.

DIV. I. ELEMENTARY PROJECTIONS.
DIV. II. DETAILS OF MASONRY, WOOD, AND METAL CONSTRUCTIONS.
DIV. III. ELEMENTARY SHADOWS AND SHADING.
DIV. IV. ISOMETRICAL AND OBLIQUE PROJECTIONS.
DIV. V. ELEMENTS OF MACHINES.
DIV. VI. SIMPLE STRUCTURES AND MACHINES.

By S. EDWARD WARREN, C.E.,

FORMER PROFESSOR IN THE RENSSELAER POLYTECHNIC INSTITUTE, MASS. INST. OF TECHNOLOGY AND BOSTON NORMAL ART SCHOOL; AND AUTHOR OF A SERIES OF TEXT-BOOKS IN DESCRIPTIVE GEOMETRY AND INSTRUMENTAL DRAWING.

THIRTEENTH EDITION.

FIFTH THOUSAND.

NEW YORK:
JOHN WILEY AND SONS,
53 EAST TENTH STREET,
1893.

PRESS OF J. J. LITTLE & CO.,
NOS. 10 TO 20 ASTOR PLACE, NEW YORK.

CONTENTS.

DIVISION VI.—SIMPLE STRUCTURES AND MACHINES.

NOTE TO THE FOURTH EDITION.

This edition is improved chiefly by the extension of the chapter on oblique, or pictorial projections, also called mechanical perspective, with the addition of a new plate.

Numerous minor corrections, and new or improved paragraphs have been scattered through the work, as suggested by further experience. Yet it is not expected that this edition, however improved, can supersede either *careful and thinking study* on the part of the willing student, or ample, *repeated, and varied* instruction on the part of the willing teacher. Many minds require variously changed and amplified statements of the same thing before they are ready to exclaim heartily, "I see it now;" and the teacher must be ready to meet, with many forms of instruction, the conditions presented to him by different minds.

A very moderate collection of such objects as are illustrated in Division II., and such as can be made by a carpenter or turner, or in machine, gas-fitting, or pattern shops, will very usefully supplement the text, and add to the pleasure and benefit derived by the student; and will be better than an increased size of the book, which is meant to be rather suggestive than exhaustive, or to be too closely followed, in respect to the examples chosen for practice. Finally, the author's chief desire in relation to this, as well as the rest of his "elementary works," is, to see them so generally used in *higher preparatory instruction* as to give due place to higher studies in the same department in the strictly Technical Schools.

R. P. I., Troy, July, 1871.

NOTE TO THE FIFTH EDITION.

THE call for a new edition of this manual has led the Author, after the lapse of ten years since the last revision, to make such further improvements in a new edition, as additional and varied experience and reflection have suggested.

A few paragraphs in Division I. have been re-written.

A few but valuable additions have been made to the text and plates of Div. II.

Div. III. contains one more plate, taken from Div. VI., and examples of finished shading, not before provided.

Div. IV. has been improved by the addition of a few figures and by partial re-writing.

The most important change is the addition of Div. V., embracing the more important and universal elements of machines. The volume is thus made more complete, both in itself, and as an introduction to higher works both theoretical and practical.*

Div. VI. (V. in previous editions) has been slightly enlarged by a few new and valuable practical examples.

In general: while the subjects of all the Author's "Elementary Works" have been largely taught in the earlier classes of Polytechnic Schools, of whatever name, it is to be hoped that by the increasing development of scientific instruction, they will all ultimately be included in *Preparatory Scientific Instruction*, and in *Special Normal Classes;* and in behalf of the many pupils whose education ends in preparatory schools, but to whom an exact knowledge of elementary instrumental or mathematical drawing would be highly useful.

The explanations of first principles have purposely been made very complete, in behalf of all classes of self-instructors, and because what is not thus printed must be said, and often repeated,

* The beautiful plates XVII., XVIII., XIX., modified however to suit a full explanatory text, are from the *Cours de dessin linéaire,* par DELAISTRE, a work which every draftsman would do well to possess.

to ensure that full understanding of the subject, the test of which is the ready performance of new examples.

At this point the testimony of an evidently experienced and *faithful* English author and teacher may well be noted. Speaking of the copying system, he says, "If, however, at the end of one or two years' practice, the copyist [though able to make a highly finished copy] is asked to make a side and end elevation and longitudinal section of his lead-pencil, or a transverse section of his instrument-box, the chances are that he can do neither the one nor the other. Strange as it may appear, this is a state of things which I have had frequent opportunities for witnessing. . . . The remedy has been to commence a *course of study from the very beginning*. . . . He has made from the copy a highly finished drawing, with all the shadows admirably projected, being at the same time, however, perfectly ignorant of the rules for projecting such shadows. This is the true picture of a student who had a course of two years' study where mechanical drawing was taught" [by merely *copying* successively more and more elaborate drawings].*

With these remarks the present edition, in its final form as now intended, is committed to the favor of Schools and Self-Instructors.

NEWTON, MASS., October, 1880.

* Preface to BINN's *Orthographic Projections*. London, 1867.

FROM THE ORIGINAL PREFACE.

EXPERIENCE in teaching shows that correct conceptions of the forms of objects having three dimensions, are obtained with considerable difficulty by the beginner, from drawings having but two dimensions, especially when those drawings are neither "natural"—that is "pictorial"—nor shaded, so as to suggest their form; but are artificial, or "conventional," and are merely "skeleton," or unshaded, line drawings. Hence moderate experience suggests, and continued experience confirms, the propriety of interposing, between the easily understood drawings of problems involving two dimensions, and the *general* course of problems of three dimensions, a *rudimentary* course upon the methods of representing objects having three dimensions.

Experience again proves, in respect to the drawing of any engineering structures that are worth drawing, that it is a great advantage to the draftsman to have—1st, *some knowledge of the thing to be drawn*, aside from his knowledge of the methods of drawing it; and 2d, *practice* in the *leisurely study* of the graphical construction of single members or elements of a piece of framing or other structure.

The truth of the second of the preceding remarks, is further apparent, from the fact that in entering at once upon the drawing of whole structures, three evils ensue, viz.—1st, *Confusion of ideas*, arising from the mass of new objects (the many different parts of a structure) thrown upon the mind at once; 2d, *Loss of time*, owing to repetition of the same detail many times in

the same structure; and 3d, *Waste of drawings*, as well as of time, through poor execution, which is due to insufficient previous practice. Hence Divisions II. and V. contain a liberal collection of elements of structures and machines, each one of which affords a useful problem, while Division VI. includes examples of a few simple structures, to fulfil the threefold purpose of affording occasion for learning the names of parts of structures; for practice in the combination of details into whole structures; and for profitable review practice in execution.

Classes will generally be found to take a lively interest in the subjects of this volume—because of their freshness to most learners, as new subjects of interesting study—because of the variety and brevity of the topics—and because of the compactness and beauty of the volume which is formed by binding together all the plates of the course, when they are well executed. As to the use of this volume, it is intended that there should be formal interrogations upon the problems in the 1st, 3d, and 4th divisions, with graphical constructions of a selection of the same or similar ones; and occasional interrogations mingled with the graphical constructions of the practical problems of the remaining divisions. Remembering that excellence in mere execution, though highly desirable and to be encouraged, is not, at this stage of the student's progress, the sole end to be attained, the student may, in place of a tedious course of finished drawings, be called on frequently to describe, by the aid of pencil or blackboard sketches, *how* he would construct drawings of certain objects—either those given in the several Divisions of this volume, or other similar ones proposed by his teacher.

PRELIMINARY NOTES.

As beginners not seldom find peculiar difficulties at the outset of the study of projections, the removal of which, however, makes subsequent progress easy, the following special explanations are here prefixed.

I. Figs. 1, 2, 3, 5, and 15, of Pl. I. are pictorial diagrams, *used for illustration in place of actual models.* Thus, in Fig. 3, for example, MH*t* represents a *horizontal* square cornered plane surface, as a floor. MGV represents a *vertical* square cornered plane surface, as a wall, which is therefore perpendicular to MH*t*. P represents any point in the angular space included by these two planes. P*p* represents a line from P, perpendicular to the plane MH*t*, and meeting it at *p*. P*p'* represents a line from P, perpendicular to the plane MGV, and meeting it at *p'*. Then P*p* and P*p'* are called the *projecting lines* of P. The point *p* is called the *horizontal projection* of P, and *p'* the *vertical projection* of P. The projecting lines of *any point or of any body*, as in Fig. 1, are perpendicular to the planes, as MH*t* and MGV, which are called *planes of projection.*

II. In preparing a lesson from this work, the object of the student is, by no means, to commit to memory the figures, but to learn, from the *first principles*, and subsequent explanations, *to see in these figures the realities in space which they represent;* so as to be able, on hearing the enunciation of any of the problems, to solve it from a clear understanding of the subject, and not "by rote" from mere memory of the diagrams. The student will be greatly aided in so preparing his lessons, by working out the problems, in space, on actual planes at right angles to each other, as on the leaves of a folding slate, when one slate is placed horizontally and the other vertically. In the construction of his *plates*, he should also test his knowledge of the *principles*, by varying the *form* of the examples, though without essentially changing their *character.*

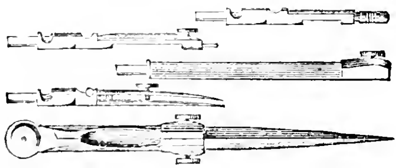
Drawing Instruments and Materials.

This volume is intended to be the immediate successor of my "Drafting Instruments and Operations," which is therefore supposed to have been read first, by students of this one.

But as some self-instructors and other students may desire to acquire a knowledge of projections as quickly as possible, for practical use, and without regard, at first, to finished execution of their drawings, the following condensed information is here inserted for their convenience.

To abridge the descriptions to the utmost, it may first be stated that dealers in Drawing Instruments and Materials are found in all large cities, who will send descriptive catalogues on application. Such are Frost & Adams, Boston; W. & L. E. Gurley, Troy, N. Y.; Keuffel & Esser, New York City; James W. Queen, Philadelphia; and others, doubtless, whose advertisements can be found in educational and popular mechanical periodicals. The necessary articles are:—

1. A good *pair of compasses*, with their *furniture;* that is, a pen, pencil, and needle point to replace the movable steel points, when drawing circles in pencil or ink.

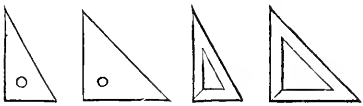


2. A good drawing pen.

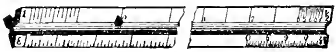
3. A drawing board 20 x 30 inches.

4. A T square; that is, a hard-wood ruler, having a stout wooden cross-piece about $2\frac{1}{2}$ x 9 inches, and half an inch thick, at one end, upon the flat side of which the blade is firmly fastened, truly at right angles. The blade may be about 30 inches long.

5. A pair of hard-wood right-angled triangles, the longest side about 10 inches long; one with the two acute angles of 45° each, the other with acute angles of 30° and 60°.



6. A triangular scale, graduated into tenths or twelfths of the unit as may be preferred; or, a flat ivory "protractor scale."



7. Buff manilla office, or "detail" paper, or, if preferred and it can be afforded, Whatman's *rough* drawing paper, of convenient size, from "medium," 17″ x 22″, to "imperial," 21″ x 30″.

8. Hard lead-pencils.

9. A cake of Indian ink—Chinese the best for shading, the Japanese for lines.

Where the utmost economy is sought, a very cheap, fair quality of brass instruments can be had in boxes, or the drawings can even be made with pencil only; any neat worker in hard wood can make the drawing board, **T** square, and triangles, and a foot-rule may be made to serve as a scale.

When drawings are not to be colored, the paper can be lightly gummed or tacked to the drawing board at the corners. Otherwise, the sheet should be well wet by sponging with clean water and, while wet, fastened to the board by means of thick mucilage applied along the edge of the paper.

Indian ink is prepared for use by rubbing it on a bit of china, with a few drops of water. It is then applied between the blades of the drawing pen by a small feather or slip of paper. Pen and ink should be wiped dry when done with.

ELEMENTARY PROJECTION DRAWING.

DIVISION FIRST.

PROJECTIONS.

CHAPTER I.

FIRST PRINCIPLES.

§ 1. *The purely Geometrical or Rational Theory of Projections.*

1. ELEMENTARY PROJECTION DRAWING is an introduction tc Descriptive Geometry, and shows how to represent *simple solids*, singly and in combination, upon plane surfaces, yet so as to show their real dimensions.

2. If ten feet of 5-inch stove-pipe were wanted, a circle five inches in diameter, drawn on paper, would be all the pattern the workman would need. But if the desired length were forgotten, or if the pipe were to be conical, the circular drawing would no longer be sufficient. That is, as a plane surface has but two dimensions, no more than two dimensions of any object can be exactly shown in one figure on that plane.

But *practical work*, and geometrical *problems for study*, are both continually arising, which require, for *convenient execution* in one case, and *proper solution* in the other, that we should be able, in some way, to truly show all the dimensions of solid bodies upon plane surfaces.

3. What, then, is the *number* and the *relative position* of the planes which will enable us to represent all the dimensions of any *geometrical solid*, in their real size, on those planes? To assist in answering this question, reference may be made to Pl. I., Fig. 1. Let ABCFED be a regular square-cornered block, whose length is AB; breadth, AD; and thickness, AC; and let MN be any horizontal plane below it and parallel to its top surface ABED. If now from the four points A, B, D and E, perpendiculars be let fall upon the horizontal plane MN, they will meet it in the points *a*, *b*, *d* and *e*.

By joining these points, it is evident that a figure—*abcd*—will be formed, which will be equal to the top surface of the block, and will be a correct representation of the *length* and *breadth* of that top surface—*i. e.* of the length and breadth of the block. Similarly, if MP be a vertical plane, parallel to the front, ABCF, of the block, and if perpendiculars, A*a′*, etc., be let fall from A, B, C, F, upon MP; the figure, *a′b′c′f′*, will be equal to ABCF, and hence will show the length and thickness of the block.

4. From the last article the following definitions arise. The point *a*, Pl. I., Fig. 1, is where A would arrive if thrown, that is, *projected*, vertically downwards along the line A*a*. Likewise, *a′* is where A would be, if thrown or *projected* along A*a′*, from A to *a′*, perpendicularly to the plane MP. Hence *a* is called a *horizontal projection* of A; and *a′* is called its *vertical projection*.

Also, conversely, A is said to be *horizontally projected* at *a*, and *vertically projected* at *a′*.

The plane MN is thence called the *horizontal plane of projection;* and the plane MP, the *vertical plane of projection.*

A*a* and A*a′* are called the *two projecting lines* of the point A relative to the *planes of projection*, MN and MP.

Hence we have this definition. If from any given point a perpendicular be let fall upon a plane, the point where that perpendicular meets the plane will be the *projection of the point upon the plane*, and the perpendicular will be the *projecting line of the point.*

5. Similar remarks apply to any number of points or to objects limited by such points; and to their projections upon any other planes of projection. Thus *abcd* is the horizontal projection of the block ABCD, and *a′b′c′f′* is its vertical projection, and, generally, the projecting lines of objects are perpendicular to the planes of projection employed. Finally, the intersection, as MR, of a *horizontal*, and a *vertical* plane of projection, is *the ground line for that vertical plane.*

6. From the foregoing articles the following principles arise. *First:* Two planes, at right angles to each other, are necessary to enable us to represent, fully, the three dimensions of a solid. *Second:* In order that those dimensions shall be seen in their true size and relative position, they must be parallel to that plane on which they are shown. *Third:* Each plane shows two of the di-

mensions of the solid, viz., the two which are parallel to it; and that dimension which is thus shown twice, is the one which is parallel to both of the planes. Thus AB, the length, and AD, the breadth, are shown on the plane MN; and AB, the length again, and AC, the thickness, are shown on the plane MP. *Fourth:* The height of the *vertical projection* of a point *above the ground line*, is equal to the *height of the point itself, in space, above the horizontal plane;* and the perpendicular distance of the *horizontal projection* of a point *from the ground line*, is equal to the *perpendicular distance of the point itself, in front of the vertical plane.* Thus: Pl. I., Fig. 1, $aa'' = Aa'$ and $a'a'' = Aa$.

7. The preceding principles and definitions are the foundation of the subject of projections, but, by attending carefully to Pl. I., Fig. 2, some useful elementary applications of them may be discovered, which are frequently applied in practice. Pl. I., Fig. 2, is a pictorial model of a pyramid, V*cdeg*, and of its two projections. The face, V*cd*, of the pyramid, is parallel to the vertical plane, and the triangle, X*ab*, is equal and parallel to V*cd*, and a little in front and at one side of it. By first conceiving, now, of the actual models, which are, perhaps, represented as clearly as they can be by mere diagrams, in Pl. I., Figs. 1 and 2; and then by attentive study of those figures, the next two articles may be easily understood.

§ II.—*Of the Relations of Lines to their Projections.*

8. *Relations of single lines to their projections.*

a. A *vertical line*, as AC, Pl. I., Fig. 1, has, for its horizontal projection, a point, *a*, and for its vertical projection, a line *a'c'*, perpendicular to the ground line, and equal and parallel to the line AC, in space.

b. A *horizontal line*, as AD, which is *perpendicular to the vertical plane*, has, for its horizontal projection, a line, *ad*, perpendicular to the ground line, and equal and parallel to the line, AD, in space; and for its vertical projection a point, *a'*.

c. A *horizontal line*, as AB, which is *parallel to both planes* of projection, has, for *both* of its projections, lines *ab* and *a'b'*, which are parallel to the ground line, and equal and parallel to the line, AB, in space.

d. A *horizontal line*, as BD, which *makes an acute angle with the vertical plane*, has, for its horizontal projection, a line, *bd*, which makes the same angle with the ground line that the line, BD,

makes with the vertical plane, and is equal and parallel to the line itself (BD); and has for its vertical projection a line $b'a'$, which is parallel to the ground line, but shorter than BD, the line in space.

e. An *oblique line*, as BC, Pl. I., Fig. 1, or Vd, Pl. I., Fig. 2, which is *parallel to the vertical plane*, has, for its vertical projection, a line $b'c'$, or $v'd'$, which is equal and parallel to itself, and for its horizontal projection, a line ba or vd, parallel to the ground line, but shorter than the line in space.

f. An *oblique line*, as Vg, Pl. I., Fig. 2, *which is oblique to both planes of projection*, has both of its projections, $v'd'$ and vg, oblique to the ground line, and shorter than the line itself.

g. An *oblique line*, as AH, Pl. I., Fig. 1, which is oblique to both planes of projection, but is *in a plane* ACDH, perpendicular to both of those planes, has both of its projections, $a'c'$ and ad, perpendicular to the ground line, and shorter than the line itself.

h. A line, lying in either plane of projection, coincides with its projection on that plane, and has its other projection in the ground line. See cd—$c'd'$, the projections of cd, Pl. I., Fig. 2.

9. *Remark.* A general principle, which it is important to be perfectly familiar with, is embodied in several of the preceding examples; viz. When any line is *parallel to either plane of projection*, its projection on *that* plane is equal and parallel to itself, and its projection *on the other plane* is parallel to the ground line.

10. The preceding remark serves to show how to find the true length of a line, when its projections are given. When the line, as Vg, Pl. I., Fig. 2, is oblique to both planes of projection, its length, Vg, is evidently equal to the *hypothenuse* of a right-angled triangle, of which the *base* is vg, the horizontal projection of the line, and the *altitude* is Vv, the height of the upper extremity, V, above the horizontal plane. When the line, as AH, Pl. I., Fig. 1, does not touch either plane of projection, it is evidently equal to the hypothenuse of a right-angled triangle, of which the base, CH, equals the *horizontal projection*, ad, and the altitude, AC, equals the *difference* of *the perpendiculars*, Aa and Hd, to the horizontal plane

In the same way, it is also true that the line, as Vg, Pl. I., Fig. 2, is the hypothenuse of another right-angled triangle, whose base equals the vertical projection, $v'd'$, and whose altitude equals the difference of the perpendiculars, Vv' and $d'g$, from the extremities of the line to the vertical plane of projection.

11. *Relations of pairs of lines to their projections.* These relations, after the full notice now given of the various positions of single lines, may be briefly expressed as follows.

a. A pair of lines which are *equal and parallel in space, and also parallel to a plane* of *projection*, as AB and CF, Pl. I., Fig. 1, or Vc and Xa, Pl. I., Fig. 2, have their projections on that plane—$a'b'$ and $c'f'$, Pl. I., Fig. 1, or $v'c'$ and $x'a'$, Pl. I., Fig. 2—*equal and parallel—to each other, and to the lines in space.*

b. A pair of lines which are *equal and parallel* in space, *but not parallel to a plane* of projection, will have their projections on that plane equal and parallel to each other, but not to the lines in space.

c. Parallel lines *make equal angles* with either plane of projection; hence it is easy to see that lines not parallel to each other—as Vd and Vc, or Vg and Ve, Pl. I., Fig. 2—but which make equal angles with the planes of projection, will have equal projections on both planes—i.e. $v'd'=v'c'$ and $vg=ve$, also $vd=vc$.

§ III.—*Physical Theory of Projections.*

12. The preceding articles comprise the substance of the purely geometrical or rational theory of projections, which, strictly, is sufficient; but it is natural to take account of the physical fact that the magnitudes in space and their representations, both address themselves to the eye, and to inquire *from what distance* and *in what direction* the magnitudes in Pl. I., Figs. 1 and 2, must be viewed, in order that they shall appear just as their projections represent them. Since the projecting lines, Q, regarded as rays, reflected from the block, Fig. 1, to the eye, are parallel, they could only meet the eye at an infinite distance in front of the vertical plane. Hence the vertical projection of an object represents it as it would appear to the eye, situated at an infinite distance from it, and looking in a direction perpendicular to the vertical plane of projection. Likewise, the projecting lines, S, show that the horizontal projection of an object represents it as seen from an infinite distance above it, and looking perpendicularly down upon the horizontal plane. Thus, *the projecting lines represent the direction of vision*, which is perpendicular to the plane of projection considered.

§ IV.—*Conventional Mode of representing the two Planes of Projection, and the two Projections of any Object upon one plane —viz. the Plane of the Paper.*

13. In practice, a single flat sheet of paper represents the two planes of projection, and in the following manner. The vertical plane, MV, Pl. I., Fig. 3, is supposed to revolve backwards, as

shown by the arcs *ru* and V*t*, till it coincides with the horizontal plane produced at M *u t* G. Hence, drawing a line from right to left across the paper, to represent the ground line, MG, all that part of the paper above or beyond such a line will represent the vertical plane of projection, and the part below it the horizontal plane of projection.

14. Elementary geometry shows that the plane, as PP′ $p''p$, Pl. I., Fig. 3, of the projecting lines, Pp and PP′, (3, 4) is perpendicular to both of the planes of projection, and to the ground line MG. Hence it intersects these planes in lines, as pp'' and P′ p'', both of which are perpendicular to the ground line at the same point p''.

15. If, now, as explained in (13) the vertical plane MV, Pl. I., Fig. 3, be revolved about MG, to coincide with the horizontal plane, the point p'' will remain in the axis MG, and the lines $p'p''$ and P′p'' will unite to form one line pp', perpendicular to MG.

That is: *Whenever two points are the projections of one point in space, the line joining them will be perpendicular to the ground line.*

§ V.—*Of the Conventional Direction of the Light; and of the Position and Use of Heavy Lines.*

16. Without going into this subject fully, as in Div. III., it is sufficient to say here that, as one faces the vertical plane of projection, the light is assumed to come from behind, and over the left shoulder, in such a direction that *each projection of a ray* (but not the ray itself) *makes an angle of 45° with the ground line*, as shown in Pl. I., Fig. 6. And note that *the light is supposed to turn with the observer*, as he turns to face any other vertical plane.

17. The *practical effect* of the preceding assumption in reference to the light, is, that upon a body of the form and position shown in Pl. I., Fig. 5, for example, the top, front, and left hand surfaces—*i. e.* the three seen in the Fig.—are illuminated, while the other three faces of the body are in the shade.

18. The *practical rule* by which the direction of the light, and its effect, are indicated in the projections, is, that all those *visible edges* of the body in space, which divide the light from the dark surfaces, are made heavy in projection.

19. To illustrate: The edges BC and CD of the body in space, Pl. I., Fig. 5, divide light from dark surfaces, and are seen in looking towards the vertical plane, and hence are made heavy in vertical projection, as seen at $b'c'$ and $c'd'$. BK and KF divide illuminated from dark surfaces, and are seen in looking towards the horizontal plane, and are therefore made heavy in the horizontal projection, as shown at bk and kf.

20. By inspection, it will be seen that the following simple rule in reference to the position of the heavy lines on the drawings, may be deduced, as an aid to the memory. In all ordinary four-sided prismatic bodies, placed with their edges respectively parallel and perpendicular to the planes of projection, *or nearly so*, the *right hand lines, and those nearest the ground line, of both projections, are made heavy.*

21. Heavy lines are of considerable use, in the case of line drawings particularly, in indicating the forms of bodies, as will be seen in future examples. In shaded drawings, the student must be careful to omit the heavy, or "*shade lines*," which habit, in making many line drawings, might lead him to add. On flat colored surfaces they should be added last, to avoid washing them, when coloring.

§ VI.—*Notation.*

22. Under the head of Notation, two points are to be considered, the manner of indicating the various lines of the diagram, and the lettering. As will be seen by examining Pl. I., Figs. 1, 2 —see V*e*, *eg*, &c.—and 5, the *visible* lines of the object represented are indicated by *full* lines; lines of construction and *invisible* lines of the object, so far as they are shown, are made in dotted lines The intersections of auxiliary planes with the planes of projection called *traces*, are represented by *broken and dotted* lines, as at PQP′, Pl. I., Fig. 16.

23. Unaccented letters indicate the *horizontal* projections of points. The same, with one or more accents, denote their *vertical* projections. The simple rule of thus *always lettering the same point with the same letter*, wherever it is shown, affords a key to every diagram, as will be shown as the course proceeds.

The projections of a body are geometrically equivalent to the

body itself, since they show its *form, position* and *dimensions.* Hence objects are considered as named by *naming their projections.* Thus, the point pp' means the point whose projections are p and p'; the line $ab—a'b'$ means the one whose projections are ab and $a'b'$. For brevity, the horizontal and vertical planes of projection are designated, respectively, as H and V.

24. In the practical applications of projections, "horizontal projections" are usually called "*plans*," and "vertical projections," "*elevations.*"

Before entering upon the study of the subsequent constructions, the terms "*perpendicular*" and "*vertical*" should be clearly distinguished. "Perpendicular" is a *relative term*, showing that any line or surface, to which it is applied, is *at right angles* to some other line or surface. "Vertical" is an *absolute term*, at any one place, and applies to any line or surface at right angles to a level, as a water surface. A vertical line, L, is perpendicular to all horizontal lines which intersect it, but if the entire system of lines thus related were inclined, so that all should be oblique, L would still be perpendicular to all the rest, though no longer vertical.

§ VII.—*Of the Use of the Method of Projections.*

25. Under this head it is to be noticed, that all drawings are made to serve one or the other of two purposes, *i.e.* they are made for *use* in aiding workmen in the construction of works; or in rendering intelligible, by means of drawings, the *real form and size* of some existing structure; or else, they are made for *ornament*, or to embellish our houses and gratify our tastes, and to show the *apparent forms and relative sizes* of objects.

26. Drawings of the former kind are often called, on account of the uses to which they are applied, "*mechanical*" or "*working*" *drawings.* Those of the latter kind are commonly called pictures; and here it is to be noticed that if "working" drawings are to show the *true*, and not the *apparent*, proportions of all parts of an object, they must, all and always, conform to this one rule, viz. *All those lines which are equal and similarly situated on the object, must be equal and similarly situated on the drawing.*

But, as is now abundantly evident, drawings made according to the method of projections, do conform to this rule; hence their use, as above described.

CHAPTER II.

PROJECTIONS OF LINES: PROBLEMS IN RIGHT PROJECTION; AND INCLUDING PROJECTIONS SHOWING TWO SIDES OF A SOLID RIGHT ANGLE.

27. The style of *execution* of the following problems is so simple, and so nearly alike for all of them, that it need not be described for each problem separately, but will be noticed from time to time. In the solution of problems, *lines are considered as unlimited, and may be produced indefinitely in either direction.*

§ I.—*Projections of Straight Lines.*

28. PROB. 1. *To construct the projections of a vertical straight line, $1\frac{1}{2}$ inches long, whose lowest point is $\frac{1}{2}$ an inch from the horizontal plane, and all of whose points are $\frac{3}{4}$ of an inch from the vertical plane.*

Remarks. *a.* The remaining figures of Pl. I. are drawn just half the size indicated by the dimensions given in the text. It may be well for the student to make them of full size.

b. Let MG be understood to be the ground line for all of the above problems, without further mention of it.

1*st.* Draw, very lightly, an indefinite line perpendicular to the ground line, Pl. I., Fig. 7.

2*d.* Upon it mark a point, a', two inches above the ground line, and another point, b', half an inch above the ground line.

3*d.* Upon the same line, mark the point a,b, three-fourths of an nch below the ground line. Then $a'\,b'$ will be the vertical, and ab the horizontal projection of the required line. (8 *a*)

29. PROB. 2. *To construct the projections of a horizontal line, $1\frac{1}{2}$ inches long, $1\frac{1}{2}$ inches above the horizontal plane, perpendicular to the vertical plane, and with its furthermost point—from the eye —$\frac{1}{4}$ of an inch from that plane.* Pl. I., Fig. 8, in connection with the full description of the preceding problem, will afford a sufficient explanation of this one.

Remark. It often happens that a diagram is made more intel-

ligible by lettering it as at *ab*, Pl. I., Fig. 7, and at *c'd'*, Pl. I. Fig 8, for thus the notation shows unmistakably, that *ab* or *c'd'* are not the projections of points but of lines.

30. PROBLEMS 3 to 8, inclusive, need now only to be enunciated with references to their constructions, in Pl. I.

Fig. 9 shows the projections of a line, $2\frac{1}{4}$ inches long, parallel to the ground line, $1\frac{1}{2}$ inches from the horizontal plane, and 1 inch from the vertical plane.

Fig. 10 is the representation of a line, 2 inches long; parallel to the horizontal plane, and 1 inch above it; and making an angle of 30° with the vertical plane.

Fig. 11 represents a line, $2\frac{1}{4}$ inches long, parallel to the vertical plane, and $1\frac{3}{4}$ inches from it, and making an angle of 60° with the horizontal plane.

Fig. 12 gives the projections of a line, $1\frac{1}{2}$ inches long, lying in the horizontal plane, parallel to the ground line, and $1\frac{1}{4}$ inches from it. The projection *a'b'* shows the line to be in H (6, *4th*).

Fig. 13 shows the projections of a line, $1\frac{1}{4}$ inches long, lying in the vertical plane, parallel to the ground line, and 1 inch from it.

Fig. 14 indicates a line, $2\frac{1}{2}$ inches long, lying in the vertical plane, and making an angle of 60° with the horizontal plane.

31. *Projections of the revolution of a point about an axis.* When a point revolves about an axis, it describes a circle, or arc, whose plane is perpendicular to the axis. Thus a point, revolving about an axis which is *perpendicular* to the vertical plane, describes an arc, *parallel* to that plane. The *vertical* projection of such an arc is an *equal arc*. Its *horizontal* projection (6) is a *straight line parallel to the ground line.*

Thus, Pl. I., Fig. 15*a*, C*a* represents a perpendicular to the vertical plane, V. The point, A, by revolving a certain distance about this axis, describes the arc AB; whose *vertical* projection is the *equal arc*, *a'b'*; and whose *horizontal* projection is *ab*, a straight line parallel to the ground line.

Likewise, briefly, in Figs. 15*b* and 15*c*, XY is a *vertical* axis. The point A, revolving about it, describes a *horizontal* arc, AB; whose *horizontal* projection, *ab*, is an *equal arc;* and whose *vertical* projection, *a'b'*, is a straight line parallel to the ground line.

32. PROB. 9. *To construct the projections of a line which is in a plane perpendicular to both planes of projection, the line being oblique to both planes of projection.* Plate I., Fig. 15, represents a

model of this problem. AB represents the line in space; *ab* its horizontal projection; *a'b'* its projection on the vertical plane MP'; and A'B' its projection on an auxiliary vertical plane PQP'; which is parallel to AB, and perpendicular to the ground line. Hence A'B'=AB.

Now in making these three planes of projection coincide with the paper, taken as the horizontal plane of projection, the plane PQP' is revolved about P'Q as an axis, till it coincides with the primitive vertical plane, MP', produced, as at P'QV'', and then the united vertical planes, MP'V'', are revolved backward about MH' as an axis into the horizontal plane. In the first revolution, A' describes, according to the last article, the horizontal arc, A'*a''*, (31) about *m* as a centre, and whose projections are *a''''a'''*, having its centre at Q, and *ma''*. Also B' describes the horizontal arc, B'*b''*, about *n* as a centre, and whose projections are *b''''b'''*, whose centre is Q, and *nb''*. Thus we see that *two or more different vertical projections*, as *a'* and *a''*, *of the same point, are in the same parallel, a'a'', to the ground line;* that is, *they are at the same height above that line.* Hence *a''* is at the intersection of *a'a''*, parallel to the ground line, MH', with *a'''a''*, perpendicular to MH'.

33. *a.* Notice further that *a''''b''''* is the horizontal projection of A'B', and that it coincides with the projection of *ab* upon PQP' Likewise, that *mn* is the vertical projection of A'B', and that it co incides with the projection of *a'b'* upon PQP'.

b. Note that B*b'*, for example (6), is equal to *bt*, and that *bt*=*b''n* the distance of the auxiliary vertical projection, *b''*, of B, from the trace, or *axis*, P'Q, of the auxiliary plane.

c. Note that *a''b''* shows the true *length* and *direction* of AB; that is, the angles made by *a''b''* with H'Q and P'Q, respectively, are equal to those made by AB with the planes of projection.

34. *To construct* Pl. I., Fig. 15, *in projection.* See Pl. I., Fig. 16, where, to make the comparison easier, like points have the same letters as in Fig. 15. Supposing the *length and direction* of the line given, we begin with *a''b''*, which suppose to be 2'' long, and to make an angle of 60° with the horizontal plane. Suppose the line in space to be 1½ inches to the left of the auxiliary vertical plane P'QP then *a' b'*, its vertical projection, will be perpendicular to the ground line, between the parallels *a''a'* and *b''b'* (32), and 1½ inches from P'Q. The horizontal projection, *ab*, will be in *a'b'* produced; *b''n*—*b'''b''''* are the two projections of the arc in which the point *b''* revolves back to its position, *n*—*b''''*, in the plane P'QP, and *b''''b*—*nb'* is the line in which *nb''''* is projected back

to its primitive position $b'b$. Therefore, b is at the intersection of $b''''b$ with $a'b'$ produced. a is similarly found, giving ab as the horizontal projection of the given line.

Art. 32 shows sufficiently how to *find* the length $a''b''$ if $ab—a'b'$ were given.

EXAMPLE. Construct the figure when a',b is the highest point.

35. *Execution.* The foregoing problems are to be inked with very black ink: the projections of given lines, and the ground line, in *heavy full* lines: and the lines of construction in *fine dotted* lines as shown in the figures. Lettering is not necessary, except for purposes of reference, as in a text book, though it affords occasion for practice in making small letters.

On the other hand, lettering, if poorly executed, disfigures a diagram so much that it should be made only after some previous practice, and then carefully; making the letters *small*, *fine*, and *regular*.

§ II.—*Right Projections of Solids.*

Remark. The term "right projection" becomes significant only when it refers to bodies which are, to a considerable extent, bounded by straight lines at right angles to each other. Such bodies are said to be drawn in right projection when their most important lines, and faces, are parallel or perpendicular to one or the other of the planes of projection.

36. PROB. 10.—*To construct the projections of a vertical right prism, having a square base; standing upon the horizontal plane, and with one of its faces parallel to the vertical plane.* Pl. II., Fig. 17.

Let the prism be 1 inch square, $1\frac{1}{2}$ inches high, and $\frac{1}{5}$ of an inch from the vertical plane.

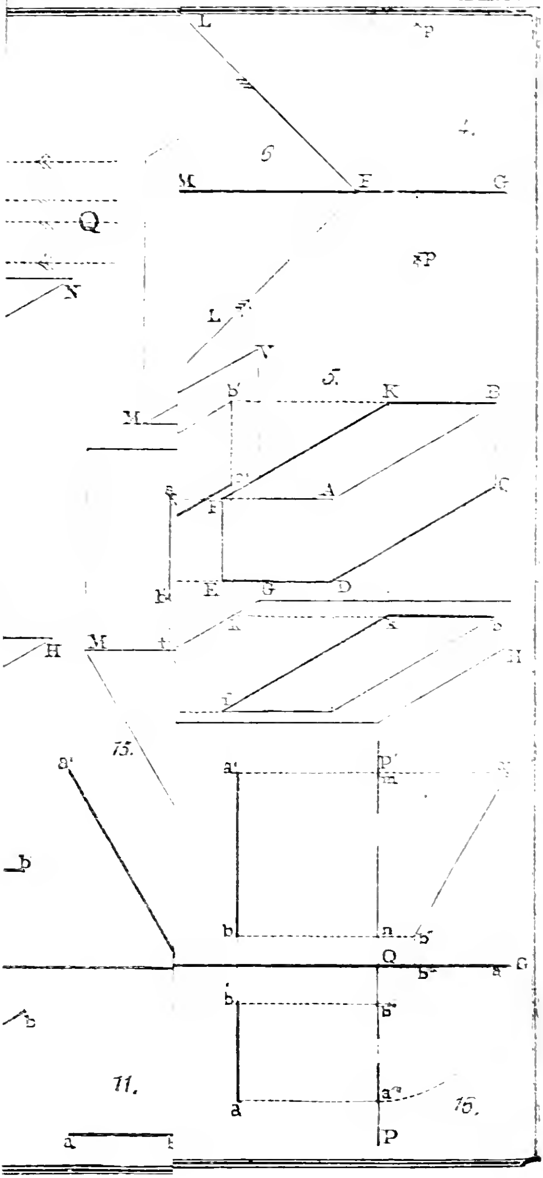
1*st.* The square ABEF, $\frac{1}{4}$ of an inch from the ground line, is the plan of the prism, and strictly represents its upper base.

2*d.* A'B'C'D', $1\frac{1}{2}$ inches high, is the elevation of the prism, and strictly represents its front face.

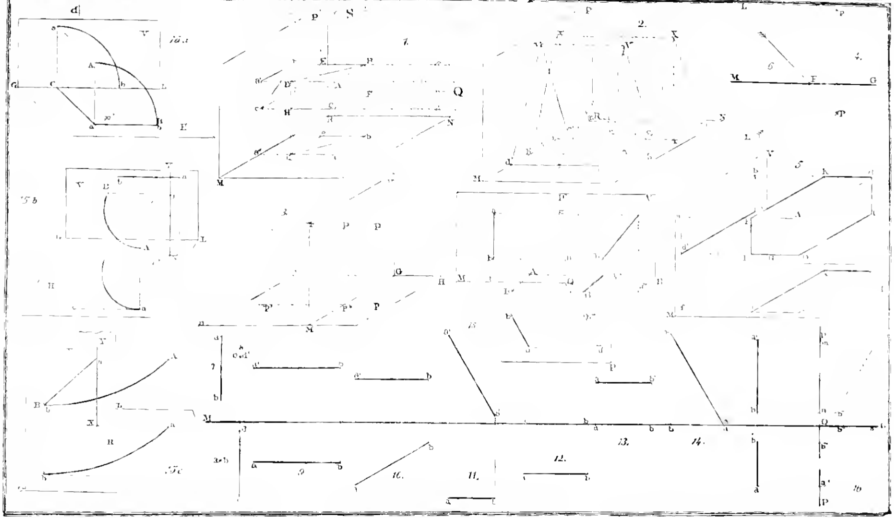
In this, and in all similar problems, it is useful to distinguish the positions of the *points, lines,* and *faces*, in words; as *upper* and *lower*; *front* and *back*; *right* and *left*; just as is done in speaking of the bodies which (26) the projections represent. Thus,

1*st.* AA' is the front, upper, left hand *corner* of the prism.

2*d.* EF—A'B' is the back top *edge*; BF—D' is the lower right hand *edge*; each corner of the plan is the *horizontal projection* of a *vertical edge*; etc.


L
M
E
G
Q
P
N
V
5.
K
B
A
C
F
E
G
D
H
M
15.
a'
p'
m
b
n
b'
O
G
b
11.
a
P
16.

PL. I.



3d. AE—A′C′ is the left hand *face;* etc.

37. Prob. 11.—*To construct the plan and two elevations of a prism having the proportions of a brick, and placed with its length parallel to the ground line.* Plate II., Fig. 18.

1st. *abcd* is the plan, $\frac{3}{4}$ of an inch broad, twice that distance in length, and $\frac{5}{8}$ of an inch from the ground line, showing that the prism in space is at the same distance from the vertical plane of projection.

2nd. $a'b'e'f'$ is the elevation, $\frac{5}{8}$ of an inch thick, and as long as the plan; and $\frac{7}{8}$ of an inch above the ground line, showing that the prism in space is at this height above the horizontal plane.

3rd. If a plane, P′QP, be placed perpendicular to both of the principal planes of projection, and touching the right hand end of the prism, it is evident that the projection of the prism upon such a plane will be a rectangle, equal, in length, to the width, *bd*, of the plan, and, in height, to the height, $b'f'$ of the side elevation. This new projection will also, evidently, be at a distance from the primitive vertical plane, *i.e.* from P′Q, equal to *d*Q, and at a distance from the horizontal plane equal to Qf'. When, therefore, the auxiliary plane, P′QP, is revolved about P′Q into the primitive vertical plane of projection, the new projection will appear at $a''e''c''g''$.

4th. dc''' is the horizontal, and $b'c''$ the vertical projection of the arc in which the point db' revolves into the primitive vertical plane. ba''', $b'a''$, are the two projections of the horizontal arc in which the corner bb' of the prism revolves.

Example.—Let the auxiliary plane PQP′ be revolved about PQ into the horizontal plane. $a''c''$ will then appear to the right of PQ and at a distance from it equal to Qb'.

38. Prob. 12.—*To construct the two projections of a cylinder which stands upon the horizontal plane.* Pl. II., Fig. 19.

The circle A*a*B*b* is evidently the plan of such a cylinder, and the rectangle A′B′C′D′ its elevation. Observe, here, that while the elevation, alone, is the same as that of a prism of the same height, Fig. 17, the plan shows the body represented, to be a cylinder.

Any point as *a* in the plan, is the horizontal projection of a vertical line lying on the convex surface, and called an *element.* A—A′C′, and B—B′D′, which limit the visible part of the convex surface, are called the extreme elements.

39. As regards execution, the right hand line B′D′ of a cylinder

or cone may be made less heavy than the line $B'D'$, Fig. 17; and in the plan, the semicircle, aBb, convex towards the ground line, and limited by a diameter ab, which makes an angle of 45° with the ground line, is made heavy, but gradually tapered, into a fine line in the vicinity of the points a and b.

40. PROB. 13.—*To construct the projections of a cylinder whose axis is placed parallel to the ground line.* Pl. II., Fig. 20.

Let the cylinder be $1\frac{1}{2}$ inches long, $\frac{3}{4}$ of an inch in diameter, its axis $\frac{3}{4}$ of an inch from the horizontal plane, and $\frac{1}{2}$ an inch from the vertical plane. The principal projections will, of course, be two equal rectangles, $gehf$ and $a'b'c'd'$, since all the diameters of the cylinder are equal. The centre lines, $g'h'$ and ab, are made at the same distances from the ground line, that the axis of the cylinder is from the planes of projection (6).

The end elevation, knowing its radius, which is equal to half of the diameter ge, or $a'c'$, of the cylinder, may be made by revolving the projection of its centre a,g', only, upon PQP', around $P'Q$ as an axis.

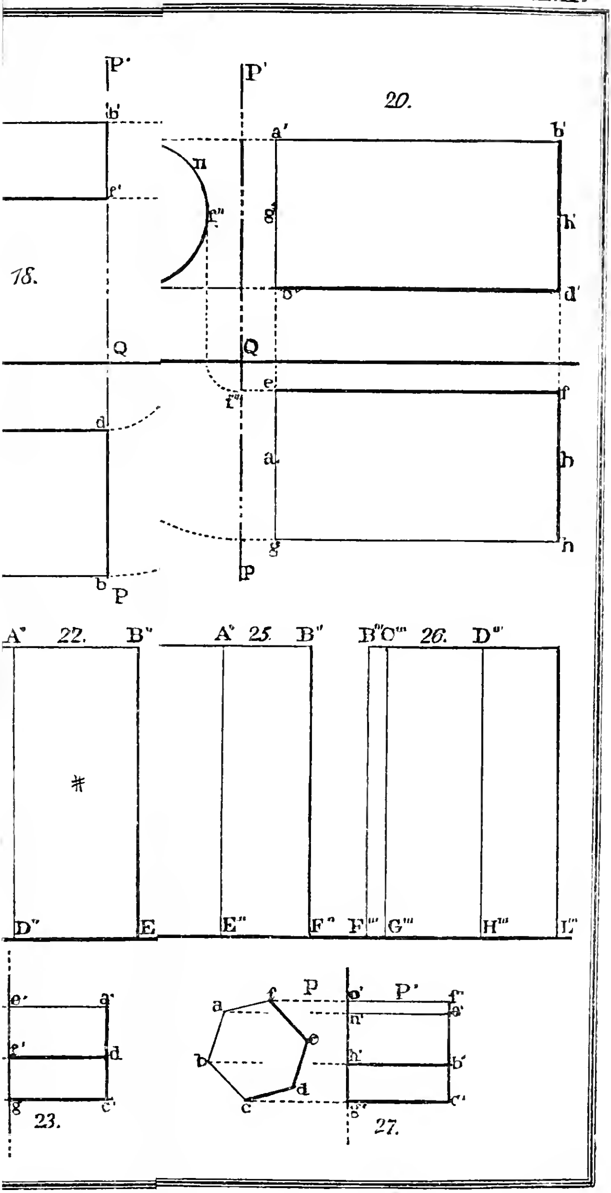
41. Standing with a *horizontal* cylinder before one, with its axis lying from right to left, and parallel to the ground line, one of its elements is its *highest* one, that is the highest above the ground, or the horizontal plane; another is the *lowest;* another, the *foremost*, that is the one nearest to one, and another the *hindmost*, or the one furthest from one. Transfering the same terms to the *projections* of the same elements, by (23) we have ab—a' b'—b'' [the three projections of] the *highest* element; ab—$c'd'$—d'', the *lowest* element; gh—$g'h'$—h'', the *foremost* element; and ef—$g'h'$—f'', the *hindmost* element.

In *inking*, the end elevation, $b''f''d''$, is made heavy at $nf''d''p$, and tapered into a fine line in the vicinity of n and p; because, by (16) when the observer turns to face the plane PQP', looking at it in the direction hg (12), the light turns with him.

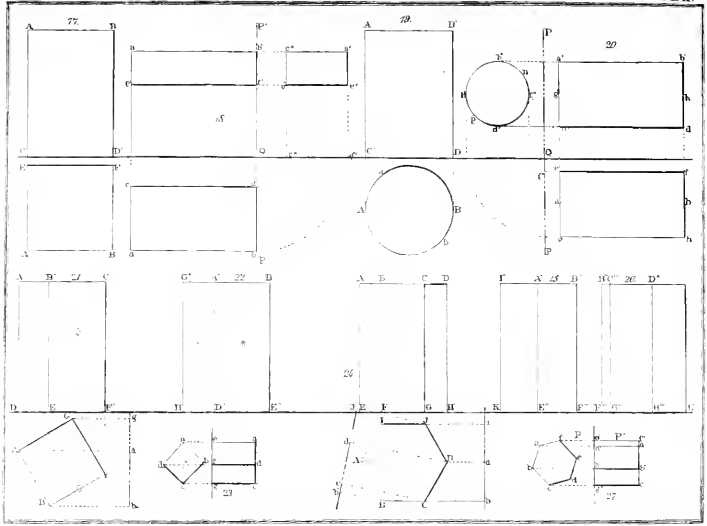
42. We have now three ways of distinguishing the projections of a horizontal *cylinder* from those of a square *prism* of equal dimensions. *First*, by *medium* instead of fully heavy lines on ef and $c'd'$. *Second*, by the lettering of the principal elements, as just explained. *Third*, and most clearly, by the circular end elevation.

§ III.—*Projections showing two sides of a Solid Right Angle.*

43. A *solid* right angle is an angle such as that at any corner of


18.
20.
22.
25.
26.
23.
27.

PL. VI.



a cube, or a square prism, and is therefore bounded by three plane right angles. When two faces of such a body are seen at once they will be seen obliquely, and neither will appear in its true size. Hence only *one* of the projections of the object will show *two* of its dimensions in their real size. Hence, we must *always make first, that projection, whichever it be, which shows two dimensions in their real size.*

44. **Prob.** 14.—*To construct the plan and two elevations of a vertical prism, with a square base; resting on the horizontal plane, and having its vertical faces inclined to the vertical plane of projection.* Pl. II., Figs. 21—22.

1*st.* ABCG is the plan, which must be made first (43) and with its sides placed at any convenient angle with the ground line.

2*d.* A′B′D′E′ is the vertical projection of that vertical face whose horizontal projection is AB.

3*d.* B′C′E′F′ is the vertical projection of that face whose horizontal projection is BC. This completes the vertical projection of the visible parts of the prism, when we look at the prism in the direction of the lines CF′, &c.

4*th.* Let *gb* be the horizontal trace of an auxiliary vertical plane of projection, which is perpendicular to both of the principal planes of projection. In looking perpendicularly towards this plane, *i.e.* in the directions G*g*, &c., AG and AB are evidently the horizontal projections of those vertical faces that would then be visible; and the projecting lines, G*g*, A*a*, and B*b* determine the widths *ga* and *ab* of those faces as seen in the new elevation. Now the auxiliary plane *gb* is not necessarily revolved about its vertical trace (not shown), but may just as well be taken up and *transferred* to any position where it will coincide with the primitive vertical plane; only its ground line *gb* must be made to coincide with the principal ground line, as at H″E″. Hence, making H″D″ and D″E″ respectively equal to *ga* and *ab*, and by drawing H″G″, &c., the new elevation will be completed.

45. The two elevations—Pl. II., Figs. 21, 22—appear exactly alike, but the faces seen in Fig. 22 are not the same as the equal ones of Fig. 21.

The different projections of the same face may be distinguished by marks. Thus the surfaces marked ⌗ are the two elevations of the same face of the prism; the one marked ϕ is visible only on the first elevation, and the one marked $\times$ is visible only on the second elevation—Fig. 22.

46. Pl. II., Fig. 23, represents a small quadrangular prism in two elevations, the axis being horizontal in space, so that the left hand elevation shows the base of the prism. In the practical applications of this construction, the centre, *s*, of the square projection is generally on a given line, not parallel to the sides of the square. Hence this construction affords occasion for an application of the problem: *To draw a square of given size, with its centre on a given line, and its sides not parallel to that line.* The following solution should be carefully remembered, it being of frequent application. Through the given centre, *s*, draw a line, L, in any direction, and another, L′, also through *s*, at right angles to L. On *each* of these lines, lay off *each way* from *s*, half the length of a side of the square. Through the points thus formed, draw lines *parallel* to the lines L and L′ and they will form the required square whose centre is *s*.

47. Prob. 15.—*To construct the plan and several elevations of a vertical hexagonal prism, which rests upon the horizontal plane of projection.* Pl. II., Figs. 24, 25, 26.

The distinction between bodies as seen perpendicularly, or obliquely, becomes obscure as we pass from the consideration of bodies whose surfaces are at right angles to each other. Figs. 24 and 25 show a hexagonal prism as much in right projection as such a body can be thus shown, but, as in both cases a majority of its surfaces are, considered separately, seen obliquely, its construction is given here.

In Fig. 24 the hexagonal prism is, as shown by the plan, placed so that two of its vertical faces are parallel to the vertical plane of projection. Observe that where the hexagon is thus placed, three of its faces will be visible, one of them in its real size, viz., BC, B′C′F′G′, and that the extreme width, E′H′, of the elevation, equals the diameter, AD, of the circumscribing circle of the plan. This is therefore the *widest* possible elevation of this prism. Notice, also, that as BC equals half of AD, while AB and CD are equal, and equally inclined to the vertical plane, the elevations, A′F′ and G′D′, of these latter faces, *will be equal, and each half as wide as the middle face.* This fact enables us to construct the elevation of a hexagonal prism situated as here described, without constructing the plan, provided we know the width and height of one face of the prism. This last construction should be remembered, it being of frequent and convenient application in the drawing of nuts, bolt-heads, &c., in machine drawing.

48. Pl. II., Fig. 25 shows the elevation of the same prism on a plane which originally was placed at *ib*, and perpendicular to the horizontal plane; whence it appears, that if a certain elevation of a hexagonal prism shows three of its faces, and one of them in its full size, another elevation, at right angles to this one, will show but two faces, neither of them in its full size; the extreme width, I″B″, of the second elevation being equal to the diameter of the inscribed circle of the plan. This is therefore the *narrowest* possible elevation of this prism.

49. Pl. II., Fig. 26 *shows the elevation of the same prism as it appears* when projected upon a vertical plane standing on *jb″*, and then transferred to the principal vertical plane, at Fig. 26. In this elevation, none of the faces of the prism are seen in their true size. The auxiliary vertical plane, on *jb″*, could have been revolved about that trace, directly back into the horizontal plane, causing the corresponding elevation to appear in the lines D*d*, &c., produced to the left of *jb″* as a ground line. Elevations on auxiliary vertical planes can always be made thus, but it seems more natural to see them side by side above the principal ground line, by transferring the auxiliary planes as heretofore described.

50. Fig. 27 represents two elevations of a hexagonal prism, placed so as to show the base in one elevation, and three of its faces, unequally, in the other. The centre of the elevation which shows the base, may be made in a given line perpendicular to *o′g′*, by placing the centre of the circumscribing circle used in constructing the hexagon, upon such a line. Having constructed this elevation, project its points, *a*,*b*, &c., across to the other vertical plane, P′, which is in space perpendicular to the plane, P, at the line, *o′g′*. By representing the elevation on P′ as touching *o′g′*, we indicate that the prism touches the plane, P, just as the elevation in Fig. 24, indicates that the prism there shown rests upon the horizontal plane.

51. Prob. 16.—*To construct the plan and two elevations of a pile of blocks of equal widths, but of different lengths, so placed as to form a symmetrical body of uniform width.* Pl. III., Figs. 28, 29.

Here for example *afg* is the plan of the lowest step; *kbe* is that of the middle step, and *cdh* that of the upper step (43).

The auxiliary vertical plane of projection, perpendicular to the horizontal plane at *k‴f‴*, is made to coincide with the principal vertical plane by direct revolution. The point *a‴a‴*, the projec

tion of aa' on the auxiliary vertical plane, revolves in a horizontal arc, of which $a'''a''$ is the horizontal, and $a''''a''$ the vertical projection (31), giving a'', a point of the second elevation. Other points of this elevation are found in the same way. This figure differs from Figs. 18 and 20, of Plate II., only in presenting more points to be constructed. If the student finds any difficulty with this example, let him refer to those just mentioned, and to first principles.

Example.—Construct an elevation on a plane parallel to *af*.

52. Prob. 17.—*To construct the vertical projection of a vertical circle, seen obliquely.* Pl. III., Fig. 30.

Let BF be the *given* projection of the circle. It is *required* to find its vertical projection, A′B′D′F′. For this purpose, the circle must be first brought into a position parallel to a plane of projection, since we can then make both of its projections, and hence can then take *both* projections of any point upon it. Let the circle be made parallel to the vertical plane. To do this, it only need be revolved about *any* vertical axis. In the figure, the axis is the vertical tangent, F—f'F′. After this revolution, the projections of the circle are bF—$b'c'$F′h'. Now taking any point on this circle, as aa', it returns about the axis F—f'F′ in the horizontal arc aA—a'A′ (31), giving A′ by projecting A upon a'A′. Likewise bb' returns in the arc bB—b'B to BB′; and cc', which is vertically under aa', returns in the arc cC—c'C′ to CC′. Thus all the points A′,B′, C′, etc., being found and joined, we have A′B′C′H′, the required oblique elevation of the vertical circle FB.

Examples.—1*st.* Let the circle be revolved about its vertical diameter HD, or any vertical axis between F and B.

2*d.* About any vertical axis; in the plane BF produced; or only parallel to it.

3*d.* Let the circle be made perpendicular to the *vertical* plane, and oblique to the horizontal plane.

53. Prob. 18.—*To construct the projections of a cylinder whose convex surface rests on the horizontal plane, and whose axis is inclined to the vertical plane.* Pl. III., Fig. 31.

As may be learned from Fig. 19, Pl. II., the projection of a right cylinder upon any plane to which its axis is parallel, will be a rectangle. Therefore let CSTV, Pl. III., Fig. 31, be the plan of the cylinder. Since it rests upon the horizontal plane, $q'u'$, in the ground line, is the vertical projection of its line of contact with that

plane, and $p'A'$ is the vertical projection of pA, the highest element of the cylinder, as it is at a height above the ground line, equal to the diameter, TV, of the cylinder. The vertical projection of either base may be found by the last problem. In the figure, the *left hand* base is thus found, and the construction, being fully given, needs no further explanation.

54. The vertical projection of the *right hand* base TV is found somewhat differently. It is revolved about its horizontal diameter, TV—T'V', till parallel to the horizontal plane. It will then appear as a circle, and a line, as $n''n$, will show the true height of n above the diameter TV. So, also, $o''o$ will show the true distance of o below TV. Therefore the vertical projections of the points n and o, will be in the line $n—n'$, perpendicular to the ground line, and at distances above and below T'V', the vertical projection of TV, equal, respectively, to nn'' and oo''. Having, in the same manner, found r' and t', the vertical projections of two points whose common horizontal projection $t—r$ is assumed, as was $n—o$, the vertical projection of the base TV can be drawn by the help of the irregular curved ruler.

55. In the *execution* of this figure, SV is made slightly heavy, and TV fully heavy, and the portion, $n'T't'$, of the elevation of the right hand base, and the small portion, $D'u'$, of the left hand base, are made heavy. Suffice it to say: *First.* That a part of the convex surface is in the light, while the right hand base is in the dark. *Second.* $n'T't'$ divides the illuminated half of the convex surface, from the base at the right, which is in the dark; and $D'u'$ divides the illuminated left hand base from the visible portion of the darkened half of the convex surface (18–20).

Example. Let the axis of the cylinder be parallel to the *vertical plane*, only.

56. Prob. 19. *To construct the two projections of a right cone with a circular base in the horizontal plane; and to construct either projection of a line, drawn from the vertex to the circumference of the base, having the other projection of the same line given.* Pl. III., Fig. 32.

Remark. When the *axis* of a cone is vertical, perpendicular to the vertical plane, or parallel to the ground line, the cone is shown in right projection as much as such a body can be, but as all the straight lines upon its surface are then inclined to one or both planes of projection, the above problem is inserted here among roblems of oblique projections.

Let VB be the radius of the circle, which, with the point V, is the horizontal projection of the cone. Since the base of the cone rests in the horizontal plane of projection, C′B′ is its vertical projection. Since the axis of the cone is vertical, V′, the vertical projection of the vertex, must be in a perpendicular to the ground line, through V, and may be assumed, unless the height of the cone is given. V′C′ and V′B′, the extreme elements, as seen in elevation, are parallel to the vertical plane of projection, hence their horizontal projections are CV and BV, parallel to the ground line (8 *e*). Let it be required to find the horizontal projection of any element, whose vertical projection, V′D′, is given. V is the horizontal projection of V′, and D′, being in the circumference of the base, is horizontally projected at D, therefore VD is the horizontal projection of that element on the front of the cone, whose vertical projection is V′D′. V′D′ is also the vertical projection of an element behind VD, on the back of the cone. Having given, VA, the horizontal projection of an element of the cone, let it be required to find its vertical projection. V′ is the vertical projection of V, and A, being in the circumference of the base, is vertically projected at A′. Therefore V′A′ is the required vertical projection of the proposed line. In inking the figure, no part of the plan is heavy lined, and in the elevation, only the element V′B′ is slightly heavy.

Examples.—1*st*. Construct three projections of a cone placed as the cylinder is in Prob. 13.

2*d*. As the cylinder is in Prob. 18.

57. Prob. 20. *To construct the projections of a right hexagonal prism; whose axis is oblique to the horizontal plane, and parallel to the vertical plane.* Pl. III., Figs. 33, 34.

1*st*. Commence by constructing the projections of the same prism as seen when standing vertically, as in Fig. 33. The plan only is strictly needed, but the elevation may as well be added here, for completeness' sake, and because some use can be made of it.

2*nd*. Draw J″G″, making any convenient angle with the ground line, and set off upon it spaces equal to G′J′, J′H′, and J′I′, from Fig. 33.

3*rd*. Since the prism is a right one, at J″, &c., draw perpendiculars to J″G″, make each of them equal to J′C′, Fig. 33, and draw F″C″, which will be parallel to J″G″, and will complete the second elevation.

4*th*. Let us suppose that the prism was moved from its first

position, Fig. 33, parallel to the vertical plane, and towards the right, and then inclined, as described, with the corner, CJ′, of the base, remaining in the horizontal plane. It is clear that all points of the new plan, as B‴, would be in parallels, as BB‴, to the ground line, through the primitive plans, as B, of the same points. It is equally true that the points of the new plan will be in perpendiculars to the ground line through the new elevations B″, &c., of the same points (15), hence these points B‴, &c., will be at th intersections of these two groups of lines. Thus, A‴ is at the intersection of AA‴ with A″A‴; C‴ is at the intersection of CC‴ with C″C‴; K‴ is at the intersection of DK‴ with H″K‴, &c.

5*th.* B‴C‴, F‴E‴, and G‴K‴, being the projections of lines of the prism which are parallel in space, are themselves parallel. A similar remark applies to C‴D‴, A‴F‴, and H‴G‴. Observe, that as the upper or visible base is viewed obliquely, it is not seen in its true size, F‴C‴ being less than FC, Fig. 33; so that this base A‴C‴, E‴, does not appear in the new plan as a regular hexagon.

58. Prob. 21. *To construct the projections of the prism, given in the previous problem, when its edges are inclined to both planes of projection.* Pl. III., Fig. 34*a*.

If the prism, Pl. III., Fig. 34, be moved to any new position, such that the inclination of its edges to the vertical plane, only, shall be changed, the inclination of its edges to the horizontal plane of projection being unchanged, the new plan will be merely a copy of the second plan, placed in a new position. Let the particular position chosen be such that the axis of the prism shall be in a plane perpendicular to the ground line, i.e. to both planes of projection; then the axis of symmetry, C‴G‴, of the second plan, will take the position C⁗G⁗, and on each side of this line the plan, Fig. 34 *a*, will be made, similar to the halves of the plan in Fig. 34.

As the prism is turned horizontally about the corner J″, and then transferred, producing the result that the inclination of its axis to the horizontal plane is unchanged, all points of the third elevation, as A⁗′, C⁗′, &c., will be in parallels to the ground line through A″, C″, &c., and in perpendiculars to the ground line, through A⁗, C⁗, &c.

By examination of this solution, and by inspection of Figs. 34 and 34*a*, it appears that a change in the position of the axis, with reference to but one plane of projection at a time, can be

represented directly from projections already given; also that a curve, beginning with the first plan, and traced through the six figures composing the three given pairs of projections in the order in which they *must* be made, would be an S curve, ending in the third elevation.

59. *Execution.*—The full explanation of the location of the heavy lines cannot here be given. The careful inquirer may be able to satisfy himself that the heavy lines of the figures, as shown, are the projections of those edges of the prism which divide its illuminated from its dark surfaces.

60. PROB. 22. *To construct the projections of a regular hexagonal pyramid, whose axis is inclined to the horizontal plane only.* Pl. III., Figs. 35, 36.

1*st.* Commence, as with the prism in the last problem, by representing the pyramid as having its axis vertical.

2*nd.* Draw $a''d''$, equal to $a'd'$, and divided in the same way. At n'', the middle point of $a''d''$, draw $n''V''$ perpendicular to $a''d''$, and make it equal to $n'V'$, which gives V'' the new elevation of the vertex. Join V'' with a'', b'', c'', and d'', and the new elevation will be completed.

3*rd.* Supposing the same translation and rotation to occur to the primitive position of the pyramid, that was made in the case of the prism (57, 4*th*), the points of the new plan, Fig. 36, will be found in a manner similar to that shown in Fig. 34. V''' is at the intersection of VV''' with $V''V'''$; c''' is at the intersection cc''' with $c''c'''$; d''' is at the intersection of dd''' with $d''d'''$, &c.

4*th.* The points, $a'''b'''c''' \ldots f'''$, of the base, are connected with V''', the new horizontal projection of the vertex, to complete the new plan. If the pyramid were less inclined, the perpendicular $V''V'''$ would fall within the base, and the whole base would then be visible in the plan. As it is, $f'''a'''$ and $a'''b'''$ are hidden, and therefore dotted.

5*th.* The heavy lines are correctly placed in the diagram; also the partially heavy lines, which are all between $V'''d'''$ and the ground line, but the reasons for their location cannot here be given, beyond the general principle (18–20) already given.

61. PROB. 23. *To construct the projections of the regular hexagonal pyramid, when its axis is oblique to both planes of projection.* Pl. III., Fig. 36*a*.

Suppose the pyramid here shown to be the one represented in

figures 35 and 36, and suppose that it has been turned about any vertical line as an axis. Then, *first*, every point of it will move *horizontally; second*, every point will hence remain at *the same height* as before; *third*, therefore, the inclination of all the edges *to the horizontal plane* will be unchanged; and hence, *fourth*, the new plan, Fig. 36*a*, will be only a copy of the second plan, Fig. 36, placed so that its axis of symmetry, $V''''d''''$, shall make any assumed angle with the ground line.

By (*second*) and (15) the points, as V'''', of the third elevation, will be at the intersection of parallels to the ground line, through the corresponding points, as V'', of the second elevation, with perpendiculars through the same points, as V'''', seen in the third plan. Observe that the two points vertically projected in c'', being at the same height above the ground line, will appear in the third elevation at c''''' and e''''', in the same straight line, through c'', and parallel to the ground line. (32).

Remembering also that lines which are parallel in space must have parallel projections, on the same plane, $c'''''d'''''$ will be parallel to $f'''''a'''''$, &c. The heavy lines are indicated in the figure.

EXAMPLE.—Construct Fig. 36*a* from Fig. 36, *without a new plan*, by taking a new vertical plane with its ground line parallel to $V''''d''''$, and revolving it directly back as mentioned in (49).

§ IV.—*Special Elementary Intersections and Developments.*

62. The positions of other planes, than those of projection, are indicated by their intersections with the planes of projection. These intersections are called *traces*.

A plane can cut a straight line in only one point; hence, if a plane cuts the ground line at a certain point, its traces, both being in the plane, must meet in that point.

In Pl. I., Fig. 5, $bBb'k'$ is a plane *perpendicular to the ground line*, MG, and, therefore, to both planes of projection, and we see that its two traces, bk' and $b'k'$, are perpendicular to the ground line at k'. Likewise in Pl. I., Fig. 15, $Aaa't$ is a plane perpendicular to the ground line MQ, and its traces at and $a't$ are perpendicular to MQ. That is: *if a plane is perpendicular to the ground line, its traces will also be perpendicular to that line.*

This is seen in regular projection, in Pl. I., Fig. 16, where PQ is the horizontal trace, and P'Q, the vertical trace, of such a plane.

In Pl. I., Fig. 5, $FKfk$ is a *plane, parallel to the vertical plane*, and it has *only a horizontal trace, fk*, which is *parallel to the ground line.* The same is true for all such planes. Likewise,

AB$a'b'$ is a *horizontal plane.* All such planes have *only a vertical trace,* as $a'b'$, *parallel to the ground line.*

In Pl. I., Fig. 2, the plane V$v'd'd$ is *perpendicular only to the vertical plane,* and, as the figure shows, the *horizontal trace only,* as dd', of such a plane, *is perpendicular to the ground line.* Also the angle $v'd'b'$, between the vertical trace, $v\ d'$, and the ground line, is the angle made by the plane with the horizontal plane.

In like manner, it can easily be seen that, if a plane be *perpendicular only to the* horizontal plane, as in case of a partly open door, its *vertical trace only* (the edge of the door at the hinges) *will be perpendicular to the ground line,* and the angle between its horizontal trace and the ground line, will be the angle made by the plane with the vertical plane of projection.

Finally, if a plane is *oblique to both planes of projection,* both of its traces will be oblique to the ground line, and at the same point. Thus, Pl. I., Fig. 6, may represent such a plane, having LF for its horizontal, and L′F for its vertical trace.

All the principles just stated can be simply illustrated by taking a book, half open, for the planes of projection, and either of the triangles for the given movable plane; and when clearly understood, the following problems can also be easily comprehended.

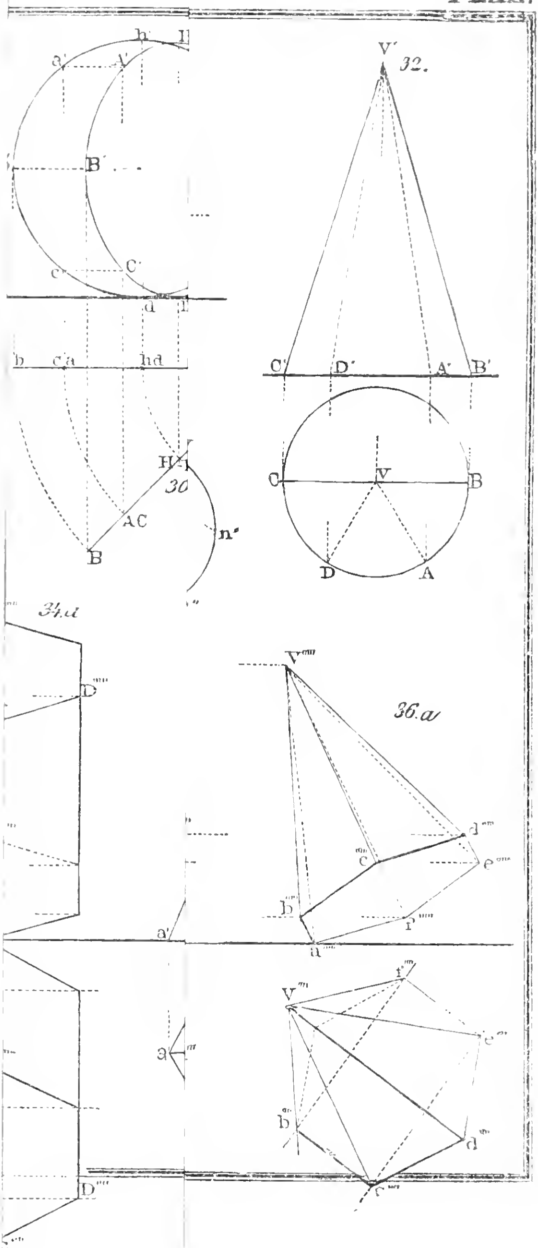
Prob. 24.—*To find the curve of intersection of a cylinder with a plane.* Pl. IV., Fig. 1.

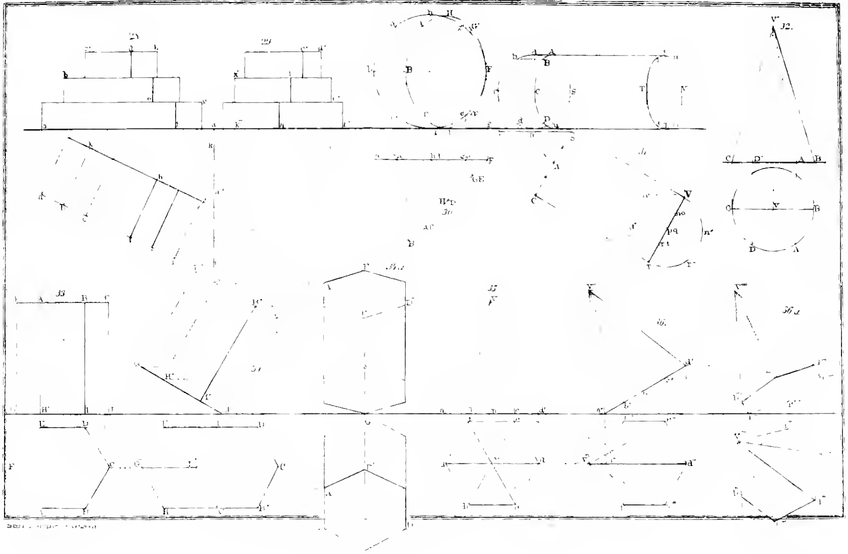
Let the cylinder, ADBG—A′B″, be vertical, and the cutting plane, PQP′, be perpendicular only to the vertical plane. All points in such a plane must have their vertical projections (that is, must be vertically projected) in the vertical trace, QP′, of the plane, but the required curve must also be embraced by the visible limits, A′A″ and B′B″, of the cylinder. Hence, $a'b'$ is the vertical projection of this curve. Again, as the cylinder is vertical, all points on its convex surface must be horizontally projected in ADBG. Hence, this circle is the horizontal projection of the required curve.

Prob. 25.—*To revolve the curve found in the last problem, so as to show its true size.*

When a plane revolves about any line in it as an axis, every point of it, not in the axis, moves in a circular arc, whose radii are all perpendicular to the axis. The representation of the revolution is much simplified by taking the axis *in, parallel to,* or *perpendicular to,* a plane of projection (31).

PL. IX.





Let AB—$a'b'$, the longer axis of the curve, and which is parallel to the vertical plane of projection, be taken as the axis of revolution. The curve may then be revolved till parallel to that plane, when its real size and form will appear. Then, at c', d', &c., the vertical projections of C and H, D and G, &c., draw perpendiculars, as $c''h''$, to $a'b'$, and make $c'c''=c'h''=n$C. Proceed likewis at d', &c., since the lines, as nC, being parallel to the horizonta plane, are seen in their true size in horizontal projection; and join the points $a'h''g''$, &c., which will give the required true form and size of the curve of intersection before found.

EXAMPLE.—This curve is an oval, called an ellipse. Its true size could have been shown by revolving its original position about DG as an axis, till parallel to the horizontal plane. The student may add this construction to the plate.

PROB. 26.—*To develope the portion of the cylinder,* Pl. IV., Fig. 1, *below the cutting plane,* PQP′.

The convex surface of a cylinder is wholly composed of straight lines, called *elements*, parallel to its axis. The convex surface of a cone is composed of similar elements, all of which meet at its vortex. Hence, each of these surfaces can evidently be rolled upon a plane, till the element first placed in contact with the plane, returns into it again. The figure, thus rolled over on the plane, is called the development of the given convex surface, and its area equals the area of that surface.

Suppose the cylinder to be hollow as if made of tin, and to be cut open along the element B′b'. Then suppose the element A′a' to be placed on the paper, as at A′a', Fig. 2, and let each half be rolled out upon the paper. The part ADB will appear to the left of A′a', and the part AGB, to the right. The base being a circle, perpendicular to the elements, will develope into a straight line B′B″, Fig. 2, found by making A′c=AC, Fig. 1, cd=CD, Fig. 1, &c., and A′h=AH, Fig. 1, &c. B′B″ may also, for convenience, be A′B′, Fig. 1, produced. Then the parallels to A′a', through c d, &c., will be developments of elements standing on C, D, &c., Fig. 1, and by projecting over upon them, a' at a', c' at c' and h'; d' at d' and g', ... B′ at b' and b'', and joining the points, the figure B′B″$b''a'b'$, will be the required development of the cylinder.

Remark.—If, now, a flat sheet of metal be cut to the patterr just found, it will roll up into a cylinder, cut off obliquely as by the plane PQP′. By making the angle P′QA′ of any desired size, the corresponding flat pattern can be made as now explained.

PROB. 27.—*To find the intersection of a vertical cone, with a plane, perpendicular to the vertical plane of projection.* Pl. IV., Fig. 3.

Let V—ADBC be the plan, and A′B′V′ the elevation of the cone, and PQ and P′Q the traces of the given cutting plane; whose horizontal trace, PQ, shows it (62) to be perpendicular to the vertical plane. For the reasons given in Problem 25, $a'b'$ will be the *vertical* projection of the required curve. The convex surface of the cone not being vertical, the *horizontal* projection of the intersection will be a curve, which must be found by constructing its points as follows.

First. The method by *elements.* Any line, as V′E′, is the vertical projection of two elements whose horizontal projections are VE and VF (Prob. 19). Therefore e', where it crosses the vertical projection, $a'b'$, of the intersection, is the vertical projection of two points of the required curve. Their horizontal projections, e and f, are found by projecting e' down upon VE and VF. Other points can be found in the same manner, except d and g, since the projecting line $d'd$ coincides with the elements VD and VG. The horizontal projections of a' and b' are a and b.

Second. The method by *circular sections.* Let M′N′ be the vertical trace of a horizontal auxiliary plane through d'. This plane will cut from the cone the circle $m'n'$—dmg, on which d' can be projected at d and g, the points desired. Other points of the horizontal projection can be found in the same manner.

Remarks.—*a.* The curve $adbg$—$a'b'$ is an ellipse whose longer axis is the line ab—$a'b'$, whose true length is $a'b'$. Its shorter axis is the line pq—p', whose true length pq bisects ab, and is always less than ab; since it is a *chord* of the circle $x'y'$ through p', and $x'y'$ is easily seen to be equal to ab. An ellipse, having thus two axes of symmetry, can be drawn by using an arc of the irregular curve that will fit one quarter of it.

b. On the cylinder, d', the middle of $a'b'$ is on the axis O—dd'. That is, *the centre of the ellipse cut from a cylinder, is on the axis of the cylinder.* Not so, however, with the cone; p', the middle of $a'b'$, is not on V′D′, the vertical projection of the axis, but is on the side of it towards the lowest point, bb', of the curve of intersection. On account of the acuteness of the intersections at p and q, these points can better be found as were d and g.

EXAMPLES.—*1st.* To make the horizontal projection less circular than in the figure, let the cone be quite flat, as at AVB, Fig. 9, and with a' near the vertex, and b' quite near the base.

2d. Find the true size of the curve by either of the ways indicated in Prob. 25, also by revolving the plane PQP′, containing it, either, about PQ as an axis, into the horizontal plane; or, about P′Q as an axis, into the vertical plane. In the former case it is only to be remembered that e'Q, for example, shows the true distance of ee' from PQ; and, in the latter case, that ek, for example, shows the true distance of ee' from the vertical trace P′Q (6).

PROB. 28.—*To develope the convex surface of a cone*, Pl. IV., Fig. 4, *together with the curve of intersection, found in the last problem.*

First. If the element VB—V′B′ be placed in contact with the paper at V′B′, and if the cone be then rolled upon the paper till this element returns into it again, as at V′B″, the development, V′B′B″, will be made. As all the elements are equal, and as the vertex is stationary, the development of the base will be the arc B′B″, with a radius equal to V′B′, the cone's slant height and of a length equal to the circumference ADBG. This length is found, as in case of the cylinder, by taking equal arcs of the base, so small that their chords shall be sensibly equal to them, and laying off those chords from B′, on the arc B′B″, till B″ is located. Thus, BE being one eighth of ADBG, its length is laid off as at B′e'' eight times to find B″.

Second. To show the curve, $adbg$—$a'b'$, on the development, consider that only the extreme elements, as VB—V′B′, show their true length in projection. Hence, the points between a' and b' must be revolved around the axis of the cone, into these elements, in order to show their true distances from the vertex. This axis being vertical, the arcs of revolution will be horizontal, and will therefore be vertically projected in the horizontal lines $c'u$, $d'n$, &c., and Vu, Vn, &c., will be the true distances of c', d', &c., from the vertex. Hence, make V′a''=V′a'; V′u', and V′u''=V′u; V′n', and V′n'' =Vn, &c., and the curve $b'a''b''$ will be the development of the intersection of the plane PQP′ with the cone.

Remark.—The remarks made upon the development of the cylinder equally apply here.

PROB. 29.—*To find the intersection of a vertical cylinder with two horizontal ones; their axes being in a plane parallel to the vertical plane of projection.* Pl. IV., Fig. 5.

ABE—A′B′A″B″ is the vertical cylinder, and MNQR—O′P′O′P″ the lower horizontal cylinder.

First. To find the highest and lowest, and foremost and hindmost points of the intersection. Since the horizontal cylinder is the smaller one, it will enter the vertical cylinder on one side, and leave it on the other, giving two curves; but as one cylinder is vertical, and the intersection, being common to both, is on it, the horizontal projections of both curves are known at once to be CAE and DBF. Now A is the horizontal projection of both the highest and lowest points of CAE. Their vertical projections are a'' and a'. Also C and E are the horizontal projections of the foremost and hindmost points, and c', on M'N', midway between O and O'', is the vertical projection of both of them. (41.)

In like manner b', b'' and d' are found.

Second. To find other intermediate points. Take the two points whose horizontal projection is G, for example. They are on the horizontal elements, one on the upper, and the other on the lower half of the horizontal cylinder, and whose horizontal projection is ST. But to find their vertical projections, we must revolve one of the bases, as MQ, till parallel to a plane of projection. Let this base revolve about its vertical diameter, O—O'O'', till parallel to the vertical plane, when OM''—O'M'''O'' will be the vertical projection of its front half. In this revolution the points, S, revolve to S'', and will thence be vertically projected at U' and S'. In counter revolution, these points return in horizontal arcs to u' and s', and $u'v'$ and $s't'$ are the vertical projections of the elements ST. Hence, project G, and also H, at g' and g'', h' and h'', and we shall have four more points of intersection. Any number of points can be similarly found.

Examples.—1*st.* The last four points could as easily have been found by revolving the base MQ about the horizontal diameter MQ—M', till parallel to the horizontal plane. This construction is left for the student.

2*d.* If the axes did not intersect each other, as at H', the points C and E would not be equidistant from OP, and would not have one point, c', for their vertical projection, and the vertical projection of the back half, as AE, of each curve would be a dotted line, separate from the same projection of the front half. The student may construct this case, also that where one of the elements, MN or QR, does not intersect the vertical cylinder.

3*d.* The horizontal cylinder, Fig. 6, shows that when the two cylinders, placed as before, are of equal diameter, the vertical projections of their curves of intersection are straight lines. Hence, each of the curves themselves is contained in a plane, that is, it is

a "*plane curve.*" This figure, if regarded separately, as a plan view, therefore may represent the plan of the intersection of two equal semi-circular arches, and the curves, KL and AY, of intersection, will be ellipses.

The curves on the cylinders in Fig. 5 cannot be contained in planes. Such curves are said to be of *double curvature.*

4th. By developing the cylinders, in Figs. 4 and 5, as in Fig. 2, the patterns may be found which will give intersecting sheet metal pipes, when rolled up in cylindrical form. The student should construct these developments, also the case in which the vertical cylinder should be the smaller one.

PROB. 30.—*To find the intersection of a horizontal cylinder with a vertical cone.* Pl. IV., Fig. 7.

Let ABV be the vertical projection of a cone, and let the circle with radius oa, be an end view of the cylinder; its axis, $o'o''$, intersecting A'V', that of the cone. Let PQ be the vertical trace of a second vertical plane, perpendicular to the ground line, as in Pl. I., Figs. 15 and 16, and let V'E'D' be the vertical projection of the cone, and G'G''N'N'' that of the cylinder, on this plane. In this construction, therefore, two vertical projections are employed, instead of a horizontal and vertical projection, for any two projections of an object are enough to show its form and position. This will more readily appear by turning the plate to bring VAB below PQ, when PQ will be the ground line, the right hand projection a plan, and the left hand one an elevation, like Fig. 5.

Now to find the intersection. Speaking as if facing the vertical plane of projection, represented by the paper to the left of PQ, after revolving that plane about PQ into the paper, AV—A'V' is the foremost element, and a' is found by projecting a across upon A'V'. Next, DV is the right hand projection of two elements, whose left hand projections are E'V' and D'V'. We therefore project G at g' and e'.

To find intermediate points. Assume any element FV, draw FF'' perpendicular to AB, then make an arc, AF'', of the plan of the cone's base, and make A'F' = A'H' = FF''. Then V'F' and V'H' will be the left hand projections of the two elements projected in FV. Then project f at f' and h' on these elements, and $g'a'h'e'$ will be the visible part of the intersection. Its right hand projection is afG, where f and G are, each, the projection of two points on opposite sides of the cone.

EXAMPLE.—By developing the cone and the cylinder, patterns could be made for a conical pipe entering a cylindrical one.

63. Observing that, in every case, the *auxiliary* planes are made to cut the *given* curved surfaces in the simplest manner, that is, in straight lines or circles, we have the following principles. To cut right lines, at once, from *two cylinders*, as in Fig. 5, a *plane must be parallel to both their axes.* To cut a *cylinder and cone,* at once, in the same manner, as in Fig. 7, *each plane must contain the vertex of the cone, and be parallel to the axis of the cylinder.* To cut elements at once from *two cones,* a plane must simply contain both vertices.

EXAMPLES.—1*st.* Thus, in Fig. 10, all planes cutting elements, both from cone VV′, and cone AA′, will contain the line VAB, hence their traces on the horizontal plane will merely pass through B. Thus the plane BD cuts from the cone, V, the elements V′*a*′—V*a*, and V′*c*′—V*c*; and from the cone, A, the elements A′D′—A*d*, and A′*d*′—AD. The student can complete the solution, the remainder of which is very similar to the two preceding.

2*d.* To find the intersection of a sphere and cone, Pl. IV., Fig. 11, auxiliary planes may most conveniently be placed in two ways. First, *horizontally.* Then each will cut a circle from the sphere, and one from the cone; whose horizontal projections will be circles, and whose intersections will be points of the intersection of the cone and sphere. Second, *vertically.* Then each plane must contain the axis of the cone, from which it will cut two elements. It will also cut the sphere in a circle, and by revolving this plane about the axis of the cone till parallel to the vertical plane, as in Prob. 17, the intersection of the circle with the revolved elements, see Prob. 27, may be noted, and then revolved back to their true position. The student can readily make the construction, after due familiarity with preceding problems has made the apprehension of the present article easy.

PROB. 31.—*To find the intersection of a vertical hexagonal prism with a sphere, whose centre is in the axis of the prism.* Pl. IV., Fig. 8.

Let O—ABC be part of the sphere, and DGHK the prism, showing one face in its real size, and therefore requiring no plan (47). Draw *dg* parallel to AC, and the arc *ehf* with O as a centre, and through *e* and *f*. This arc is the real size of the intersection of the middle face of the prism with the surface of the sphere. All the faces, being equal, have circular tops, equal to *ehf*; but, being

seen obliquely, they would be really elliptical in projection. It is ordinarily sufficient, however, to represent them by circular arcs, tangent to *hn*, the horizontal tangent at *h*, and containing the points *d* and *e*, and *f* and *g*, as shown.

Remark.—The heavy lines here, show the part of the prism within the sphere, as a spherical topped bolt head. To make D*d*=EF, draw O*d* at 45° with AC, to locate *dg*. To make the spherical top flatter, for the same base DG, take a larger sphere, and a plane above its centre for the base of the prism.

Prob. 32.—*To construct the intersection of a vertical cone with a vertical hexagonal prism; both having the same axis.* Pl. IV., Fig. 9.

Let VAB be the cone, and CFGH, the prism, whose elevation can be made without a plan (48), since one face is seen in its real size. The semicircle on *cf* is evidently equal to that of the circumscribing circle of the base of the prism, and *ct* is the chord of two thirds of it. Then half of *ct*, laid off on either side of O, the middle of CF, as at O*n*, will give *np*.the projection of the middle face ED*d* after turning the prism 90° about its axis. This done, *np* will be the height, above the base, of the highest point at which this and all the faces will cut the cone. A vertical plane, not through the vertex of a cone, cuts it in the curve, or "conic section," called a *hyperbola.* The vertical edges of the prism cut the cone at the height F*f*, hence, drawing the curves, as *dse*, sharply curved as at *s*, and nearly straight near *d* and *e*, we shall have a sufficiently exact construction of the required intersection.

Remark.—The heavy lines represent the part of the prism within the cone, finished as a hexagonal head to an iron "bolt," such as is often seen in machinery. The horizontal top, *hg*, of the head, may be drawn by bisecting *pr* at *g*. To make C*e*=ED, as is usual in practice, simply draw O*c* at an angle of 45° with AB, to locate *cf* By making VAB=30° perhaps the best proportions will be found.

64. In the subsequent applications of projections in practical problems, the ground line is very generally omitted; since a knowledge of the object represented makes it evident, on inspection, which are the plans, and which the elevations.

General Examples.

The careful study of the detailed explanations of the preceding problems, will enable the student to solve any of the following additional examples.

Ex. 1.—In Prob. 24, substitute for the cylinder any prism, find the intersection with the plane PQP′, and, by Prob. 25, find the true form and size of this intersection.

Ex. 2.—In Prob. 27, substitute for the cone any pyramid. Vary this and Ex. 1 by different positions of PQP′, cutting *both bases* in Ex. 1.

Ex. 3.—In Ex. 2, find, by Prob. 25 or by Prob. 27, Ex. 2*d*, the true form and size of the intersection and, by Prob. 28, the development of the convex surface of the pyramid.

Ex. 4.—In Probs. 22, 23, substitute for the pyramid a cone whose convex surface, rolling on H (23), shall be shown, first, with its axis parallel to V; and, second, with its axis oblique to V.

Ex. 5.—In Ex. 4, find the intersection of the cone with any plane parallel to H; and show the curve on both positions of the cone.

Ex. 6.—In Ex. 5, let the cutting plane be vertical but oblique to V, and not containing the cone's vertex.

Ex. 7.—In Prob. 29, let the horizontal cylinder be the larger one, and, after finding its intersection with the vertical one, develope it.

Ex. 8. In Probs. 22, 23, substitute for the pyramid a cylinder.

Ex. 9.—In Probs. 22, 23, substitute for the pyramid a hollow hemisphere.

Ex. 10.—In Prob. 29, let the axis of the horizontal cylinder be inclined first to H only, and then to both H and V.

Ex. 11.—In Probs. 22, 23, let the pyramid, when in the position shown in Fig. 36 (but more inclined), rest its edge $V''a''$ against an upper edge of a cube standing on H.

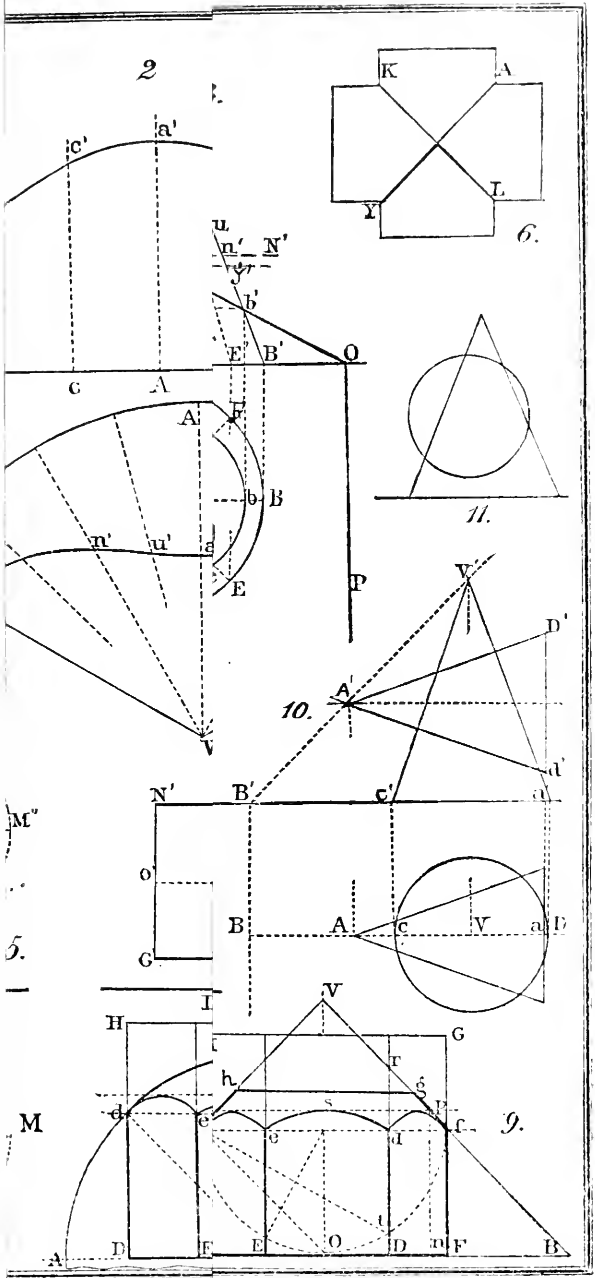
Ex. 12.—Find the four following sections of a sphere: one by a horizontal plane, one by a plane parallel to V, one by a vertical plane oblique to V, and one by a plane perpendicular to V and oblique to H.

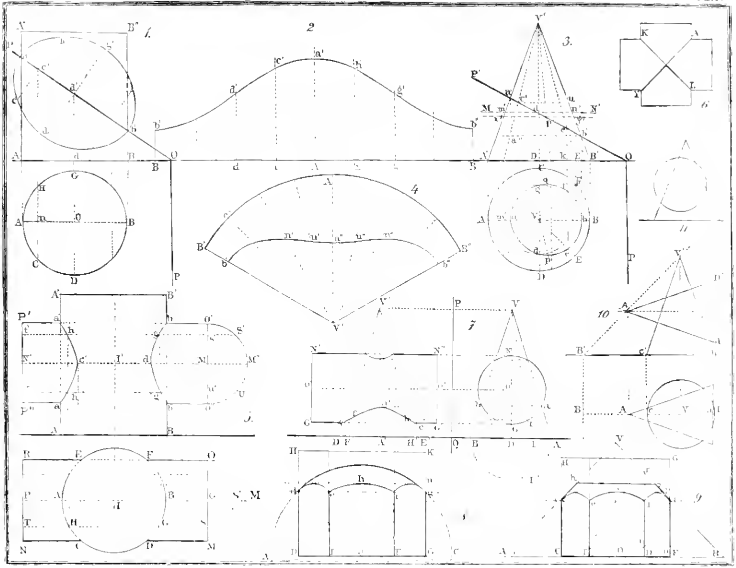
Ex. 13.—Cut a regular hexagon from a cube.

Ex. 14.—Cut a rhombus and an isosceles triangle from the square prism. Pl. II., Fig. 17.

Ex. 15.—Construct the projections of the cylinder, Pl. IV., Fig. 1, after rotating it and PQP′, together, 45° on its axis.

Ex. 16.—Substitute for the blocks, Pl. III., Figs. 28, 29, a pile of thin cylinders of unequal diameters, but with a common axis placed obliquely to V.


2
c'
a'
c
A
u
n'
N'
y'
b'
E'
B'
O
F
b
B
n'
u'
E
P
K
A
L
Y
6.
11.
V'
D'
A'
10.
d'
N'
B'
c'
a
M"
o'
B
A
c
V
a
D
5.
G
V
H
I
G
r
h
g
M
d
e
s
f
9.
e
d
t
A
D
E
E
O
D
n
F
B



DIVISION SECOND.

DETAILS OF MASONRY, WOOD, AND METAL CONSTRUCTIONS.

CHAPTER I.

CONSTRUCTIONS IN MASONRY.

§ I.—*General Definitions and Principles applicable both to Brick and Stone-work.*

65. A horizontal layer of brick, or stone, is called a *course.* The seam between two courses is called a *coursing-joint.* The seam between two stones or bricks of the same course, is a *vertical* or *heading-joint.* The vertical joints in any course should abut against the solid stone or brick of the next courses above and below. This arrangement is called *breaking joints.* The particular arrangement of the pieces in a wall is called its bond. As far as possible, stones and bricks should be laid with their *broadest surfaces horizontal.* Bricks or stones, whose length is in the direction of the length of a wall, are called *stretchers.* Those whose length is in the direction of the thickness of a wall, are called *headers.*

§ II.—*Brick Work.*

66. If it is remembered that bricks used in building have, usually, n invariable size, $8'' \times 4'' \times 2''$ (the accents indicate inches), and hat in all ordinary cases they are used whole, it will be seen that brick walls can only be of certain thicknesses, while, in the use of stone, the wall can be made of any thickness.

Thus, to begin with the thinnest house wall which ever occurs, viz. one whose thickness equals the length of a brick, or 8 inches; the next size, disregarding for the present the thickness of mortar, would be the length of a brick added to the width of one, or equal to the width of three bricks, making 12 inches, a thickness employed in the partition walls and upper stories of first class houses, o the

outside walls of small houses. Then, a wall whose thickness is equal to the length of two bricks or the width of four, making 16 inches, a thickness proper for the outside walls of the lower stories of first class houses; and lastly, a wall whose thickness equals the length of two bricks added to the width of one; or, equals the width of five bricks, or 20 inches, a thickness proper for the basement walls of first class houses, for the lower stories of few-storied, heavy manufactory buildings, &c.

67. In the common bond, generally used in this country, it may be observed—

a. That in heavy buildings a common rule appears to be, to have one row of headers in every six or eight rows of bricks or courses, i.e. five or seven rows of stretchers between each two successive rows of headers; and,

b. That in the 12 and 20 inch walls there may conveniently be a row of headers in the back of the wall, intermediate between the rows of headers in the face of the wall, while in the 8 inch and 16 inch walls, the single row of headers in the former case, and the double row of headers in the latter, would take up the whole thickness of the wall, and there might be no intermediate rows of headers.

c. The separate rows, making up the thickness of the wall in any one layer of stretchers, are made to break joints in a horizontal direction, by inserting in every second row a half brick at the end of the wall.

68. Calling the preceding arrangements common bonds, let us next consider the bonds used in the strongest engineering works which are executed in brick. These are the *English bond* and the *Flemish bond.*

The English Bond.—In this form of bond, every second course, as seen in the face of the wall, is composed wholly of headers, the intermediate courses being composed entirely of stretchers. Hence, in any practical case, we have given the thickness of the wall and the arrangement of the bricks in the front row of each course, and are required to fill out the thickness of the wall to the best advantage.

The Flemish Bond.—In this bond, each single course consists of alternate headers and stretchers. The centre of a header, in any course, is over the centre of a stretcher in the course next above or below. The face of the wall being thus designed, it remains, as before, to fill out its thickness suitably.

69. EXAMPLE 1. **To represent an Eight Inch Wall in English Bond.** Let each course of stretchers consist of two rows, side

by side, the bricks in which, break joints with each other horizontally. Then the joints in the courses of headers, will be distant half the width of a brick from the vertical joints in the adjacent courses of stretchers, as may be at once seen on constructing a diagram.

70. Ex. 2. **To represent a Twelve Inch Wall in English Bond.** See Pl. V., Fig. 37. In the elevation, four courses are shown. The upper plan represents the topmost course, and in the lower plan, the second course from the top is shown. The courses having stretchers in the face of the wall, could not be filled out by two additional rows of stretchers, as such an arrangement would cause an unbroken joint along the line, *ab*, throughout the whole height of the wall—since the courses having headers in the face, *must* be filled out with a single row of stretchers, in order to make a twelve inch wall, as shown in the lower plan.

In order to allow the headers of any course to break joints with the stretchers of *the same* course, the row of headers may be filled out by a brick, and a half brick—split lengthwise—as in the upper plan; or by two three-quarters of bricks, as seen in the lower plan.

71. Ex. 3. **To represent a Sixteen Inch Wall in English Bond.** The simplest plan, in which the joints would overlap properly, seems to be, to have every second course composed entirely of headers, breaking joints horizontally, and to have the intermediate courses composed of a single row of stretchers in the front and back, with a row of headers in the middle, which would break joints with the headers of the first named courses. If the stretcher courses were composed of nothing but stretchers, there would evidently be an unbroken joint in the middle of the wall extending through its whole height.

72. Ex. 4. **To represent an Eight Inch Wall in Flemish Bond.** Pl. V., Fig. 38, shows an elevation of four courses, and the plans of two consecutive courses. The general arrangement of both courses is the same, only a brick, as AA′, in one of them, is set six inches to one side of the corresponding brick, B, of the next course —measuring from centre to centre.

73. Ex. 5. **To represent a Twelve Inch Wall in Flemish Bond.** Pl. V., Fig. 39, is arranged in general like the preceding figures, with an elevation, and two plans. One course being arranged as indicated by the lower plan, the next course may be made up in two ways, as shown in the upper plan, where the grouping shown at the right, obviates the use of half bricks in every second course.

There seems to be no other simple way of combining the bricks in this wall so as to avoid the use of half bricks, without leaving open spaces in some parts of the courses.

74. Ex. 6. **To represent a Sixteen Inch Wall in Flemish Bond.** Pl. V., Fig. 40. The figure explains itself sufficiently. Bricks may not only be split crosswise and lengthwise, but even thicknesswise, or so as to give a piece $8 \times 4 \times 1$ inches in size. Although, as has been remarked, whole bricks of the usual dimensions can only form walls of certain sizes, yet, by inserting fragments, of proper sizes, any length of wall, as between windows and doors, or width of pilasters or panels, may be, and often is, constructed. By a similar artifice, and also by a skilful disposition of the mortar in the vertical joints, tapering structures, as tall chimneys, are formed.

§ III.—*Stone Work.*

75. The following examples will exhibit the leading varieties of arrangement of stones in walls.

EXAMPLE 1. **Regular Bond in Dressed Stone.** Pl. VI., Fig. 41. Here the stones are laid in regular courses, and so that the middle of a stone in one course, abuts against a vertical joint in the course above and the course below. In the present example, those stones whose ends appear in the front face of the wall, seen in elevation, take up the whole thickness of the wall as seen in plan.

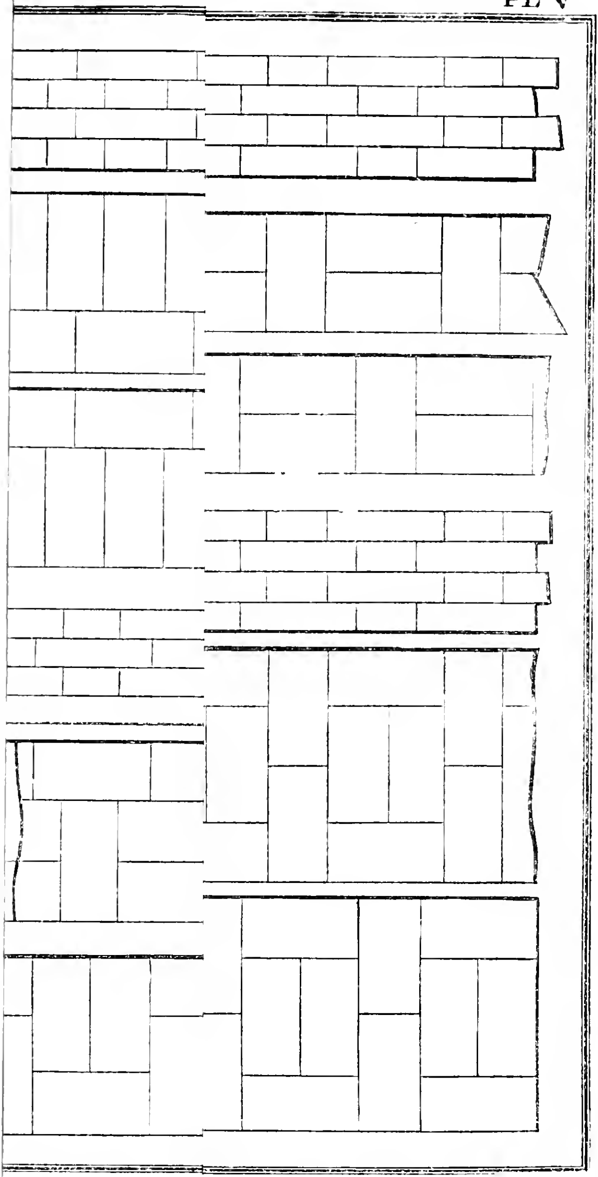
The right hand end of the wall is represented as broken down in all the figures of this plate. Broken stone is represented by a smooth broken line, and the under edge of the outhanging part of any stone, as at n, is made heavy.

76. Ex. 2. **Irregular Rectangular Bond.** Pl. VI., Fig. 42. In this example, each stone has a rectangular face in the front of the wall. These faces are, however, rectangles of various sizes and proportions, but arranged with their longest edges horizontal, and also so as to break joints.

77. That horizontal line of the plan which is nearest to the lower border of the plate, is evidently the plan of the top line of the elevation, hence all the extremities, as a', b', &c., of vertical joints, found on that line, must be horizontally projected as at a and b, in the horizontal projection of the same line.

78. Ex. 3. **Rubble Walls.** The remaining figures of Pl. VI., represent various forms of "rubble" wall. Fig. 43 represents a wall of broken boulders, or loose stones of all sizes, such as are found abundantly in New England. Since, of course, such stones

PL. V



PL. V

would not fit together exactly, the "chinks" between them are filled with small fragments, as shown in the figure. Still smaller irregularities in the joints, which are not thus filled, are represented after tinting by heavy strokes in inking. Fig. 44 represents the plan and elevation of a rubble wall made of slate; hence, in the plan, the stones appear broad, and in the elevation, long and thin, with chink stones of similar shape. Fig. 45 represents a rubble wall, built in regular courses, which gives a pleasing effect, particularly if the wall have cut stone corners, of equal thickness with the rubble courses.

Ex. 4. **A Stone Box-culvert.** Pl. VI., Figs. C, D, E. Scale $\frac{3}{16}$ of an inch to 1 ft.

Fig. C is a longitudinal section; D, part of an end elevation; and E, part of a transverse section. Waste water flowing over the dam dd', into the well a between the wing-walls a and b' and the head h, escapes by the culvert cc--c'', which is strengthened by an intermediate cross-wall m'', occurring in the course of its length.

The masonry rests on a flooring of 2-inch planks lying transversely on longitudinal sills, which, in turn, rest on transverse sills. Thus a firm continuous bearing is formed which prevents unequal settling of the masonry, while washing out underneath is provided against by sheet piling partly shown at p, p', p'', and extending six feet into the ground.

The student should construct this example on a larger scale, from 4 to 6 sixteenths of an inch to a foot; and should add a plan, or a horizontal section, both of which may easily be constructed from the data afforded by the given figures.

79. *Execution.*—Plate VI. may be, 1*st*, pencilled; 2*d*, inked in fine lines; 3*d*, tinted. The rubble walls, having coarser lines for the joints, may better be tinted, before lining the joints in ink.

Also, in case of the rubble walls, sudden heavy strokes may be made occasionally in the joints, to indicate slight irregularities in their thickness, as has already been mentioned.

The right hand and lower side of any stone, not joining another stone on those sides, is inked heavy, in elevation, and on the plans as usual. The left-hand lines of Figs. 43 and 44 are

tangent at various points to a vertical straight line, walls, such as are represented in those figures, being made vertical, at the finished end, by a plumb line, against which the stones rest.

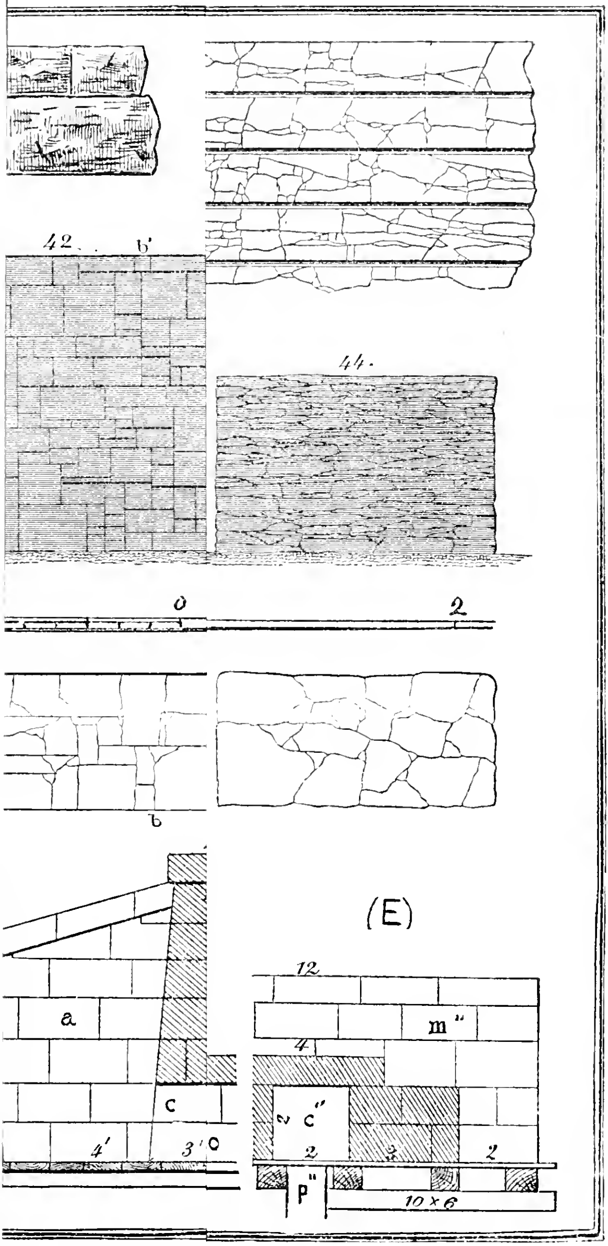
The shaded elevations on Pl. VI. may serve as guides to the *depth* of color to be used in tinting stone work. The tint actually to be used, should be very light, and should consist of gray, or a mixture of black and white, tinged with Prussian blue, to give a blue gray, and carmine also if a purplish gray is desired.

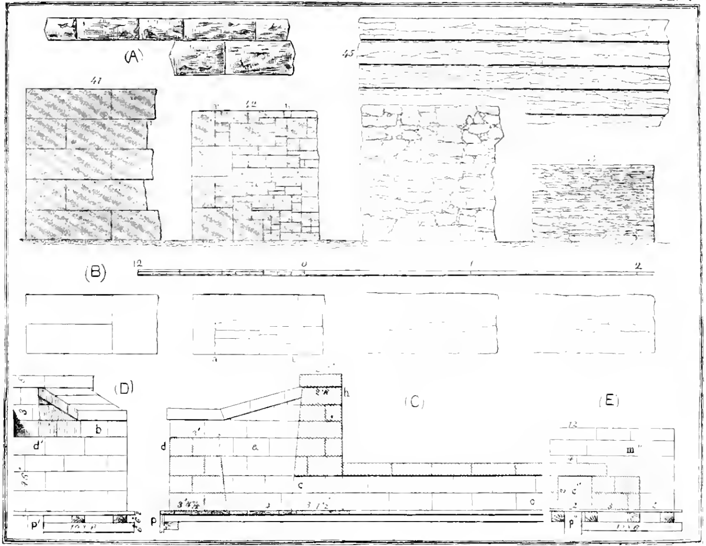
Remarks.—a. A scale may be used, or not, in making this plate. The number of stones shown in the width of the plans, shows that the walls are quite thick.

b. Rubble walls, not of slate, are, strictly, of two kinds: first, those formed of small boulders, used whole, or nearly so; and second, those built of broken rock. Each should show the broadest surfaces in *plan*.

c. After tinting, add pen-strokes, called hatchings, to represent the character of the surface; as in Fig. A., for rough, or undressed stone: in waving rows from left to right of *short, fine, equal, vertical* strokes, for smooth stone; and in a mixture of numerous fine dots and small angular marks, for a finely picked-up surface. (See actual stone work, good drafting copies, and my "Drafting Instruments and Operations.")

PL.VI.




(A)
(B)
(C)
(D)
(E)

CHAPTER II.

CONSTRUCTIONS IN WOOD

§ I.—*General Remarks.*

80. Two or more beams may be framed together, so as to make any angle with each other, from 0° to 180°; and so that the plane of two united pieces may be vertical, horizontal, or oblique.

81. To make the present graphical study of framings more fully rational, it may here be added, that pieces may be framed with reference to resisting forces which would act to separate them in the direction of any one of the three dimensions of each. Following out the classification in the preceding article, let us presently proceed to notice several examples, some mainly by general description of their *material construction* and *action*, and some by a complete description of their *graphical construction* and *execution*, also.

82. Two other points, however, may here be mentioned. *First:* A pair of pieces may be *immediately* framed into each other, or they may be *intermediately* framed by "bolts," "keys," &c., or both modes may be, and often are, combined. *Second:* Two combinations of timbers which are alike in general appearance, may be adapted, the one to resist extension, and the other, compression, and may have slight corresponding differences of construction.

83. *Note.*—For the benefit of those who may not have had access to the subject, the following brief explanation of scales, &c., is here inserted. (See my "Drafting Instruments and Operations.")

Drawings, showing the pieces as taken apart so as to show the mode of union of the pieces represented, are called "*Details.*"

Sections, are the surfaces exposed by cutting a body by planes, and, strictly, are *in* the planes of section.

Sectional elevations, or plans, show the parts both *in*, and *beyond*, the planes of section.

Drawings are made in plan, side and end elevations, sections and details, or in as few of these as will show clearly all parts of the object represented.

84. In respect to the instrumental operations, these drawings are

supposed to be "made to scale," from measurements of models, or from assumed measurements. It will, therefore, be necessary, before beginning the drawings, to explain the manner of sketching the object, and of taking and recording its measurements.

85. In sketching the object, make the sketches in the same way in which they are to be drawn, i.e. *in plan and elevation*, and not in perspective, and make enough of them to contain all the measurements, i.e. to show all parts of the object.

In measuring, take measurements of all the parts which are to be shown; and not merely of individual parts alone, but such connecting measurements as will locate one part with reference to another.

86. The usual mode of recording the measurements, is, to indicate, by arrow heads, the extremities of the line of which the figures between the arrow heads show the length.

87. For brevity, an accent (′) denotes feet, and two accents (″) denote inches. The dimensions of small rectangular pieces are indicated as in Pl. VII., Fig. 50, and those of small circular pieces, as in Fig. 51.

88. In the case of a model of an ordinary house framing, such as it is useful to have in the drawing room, and in which the sill is represented by a piece whose section is about $2\frac{1}{2}$ inches by 3 inches, a scale of one inch to six inches is convenient. Let us then describe this scale, which may also be called a scale of two inches to the foot.

The same scale may also be expressed as a scale of one foot to two inches, meaning that one foot on the object *is represented by* two inches on the drawing; also, as a scale of $\frac{1}{6}$, thus, a foot being equal to twelve inches, 12 inches on the object is represented by two inches on the drawing; therefore, *one* inch on the drawing represents *six* inches on the object, or, *each* line of the drawing is $\frac{1}{6}$ of the same line, as seen upon the object; each *line*, for we know from Geometry that *surfaces* are to each other as the squares of their homologous dimensions, so that if the length of the *lines* of the drawing is *one-sixth* of the length of the same lines on the object, the *area* of the drawing would be *one thirty-sixth* of the area of the object, but the scale always refers to the relative *lengths* of the *lines* only.

89. In constructing the scale above mentioned, upon the stretched drawing paper, see Pl. VI., Fig. B

1*st*. Set off upon a fine straight pencil line, two inches, say three times, making four points of division.

2d. Number the left hand one of these points, 12, the next, 0, the next, 1, the next, 2, &c., for additional points.

3d. Since each of these spaces represents a foot, if any one of them, as the left hand one, be divided into twelve equal parts, those parts will be representative inches. Let the left hand space, from (12) to (0) be thus divided, by fine vertical dashes, into twelve equal parts, making the three, six, and nine inch marks longer, so as to catch the eye, when using the scale.

4th. As some of the dimensions of the object to be drawn are measured to quarter inches, divide the first and sixth of the inches, already found, into quarters; dividing two of them, so that each may be a check upon the other, and so that there need be no con tinual use of one of them, so as to wear out the scale.

5th. When complete, the scale may be inked; the length of it in fine parallel lines about $\frac{1}{20}$ of an inch apart.

90. It is now to be remarked that these spaces are always to be called by the names of the dimensions they represent, and not according to their actual sizes, i. e. the space from 1 to 2 represents a foot upon the object, and is called a foot; so each twelfth of the foot from 12 to 0 is called an inch, since it represents an inch on the object; and so of the quarter inches.

91. Next, is to be noticed the directions in which the feet and inches are to be estimated.

The feet are estimated from the zero point towards the right, and the inches from the same point towards the left.

Thus, to take off 2′—5″ from the scale, place one leg of the dividers at 2, and extend the other to the fifth inch mark beyond 0, to the left; or, if the scale were constructed on the edge of a piece of card-board, the scale being laid upon the paper, and with its graduated edge against the indefinite straight line on which the given measurement is to be laid off, place the 2′ or the 5″ mark, at that point on the line, from which the measurement is to be laid off, according as the given distance is to be to the left or right of the given point, and then with a needle point mark the 5″ point or the 2′ point, respectively, which will, with the given point, include the required distance.

92. Other scales, constructed and divided as above described, only smaller, are found on the ivory scale, marked 30, &c., meaning 30 feet to the inch when the tenths at the left are taken as feet; and meaning three feet to the inch when the larger spaces —three of which make an inch—are called feet, and the twelfths of the left hand space, inches. Intermediate scales are marked

35, etc. Thus, on the scale marked 45, four and a half of the larger spaces make one inch, and the scale is therefore one of four and a half feet to one inch, when these spaces represent feet; and of forty-five feet to one inch, when the tenths represent feet. In like manner the other scales may be explained.

So, on the other side of the ivory, are found scales marked $\frac{5}{8}$, &c., meaning scales of $\frac{5}{8}$ inch to *one* foot, or *ten* feet, according as the whole left hand space, or its tenth, is assumed as representing one foot. *Note* that $\frac{5}{8}$ of an inch to a foot is $\frac{8}{5}$ of a foot to the inch, $\frac{5}{8}$ of an inch to *ten* feet, is 16 feet to an inch, &c.

94. Of the immense superiority of drawing by these scales, over drawing without them, it is needless to say much: without them, we should have to go through a mental calculation to find the length of every line of the drawing. Thus, for the piece which is two and a half inches high, and drawn to a scale of two inches to a foot, we should say—$2\frac{1}{2}$ inches$=\frac{2\frac{1}{2}}{12}$ of a foot$=\frac{5}{24}$ of a foot. One foot on the object=two inches on the drawing, then $\frac{1}{24}$ of a foot on the object$=\frac{1}{24}$ of 2 inches$=\frac{2}{24}=\frac{1}{12}$ of an inch, and $\frac{5}{24}$ of a foot ($=2\frac{1}{2}$ inches)$=\frac{5}{24}$ of 2 inches$=\frac{5}{12}$ of an inch.

A similar tedious calculation would have to be gone through with for every dimension of the object, while, by the use of scales, like that already described, we take off the same number of the feet and inches of the scale, that there are of real feet and inches in any given line of the object.

§ II.—*Pairs of Timbers whose axes make angles of* 0° *with each other.*

The student should be required to vary all of the remaining constructions in this DIVISION, in one or more of the following ways. *First*, by a change of *scale; Second*, by choosing *other examples* from models or otherwise, but of similar character; or, *Third*, by a change in the *number and arrangement of the projections* employed in representing the following examples.

95. EXAMPLE 1. **A Compound Beam bolted.** Pl. VII., Fig. 46. *Mechanical Construction.*

The figure represents one beam as laid on top of another. Thus situated, the upper one may be slid upon the lower one in the direction of two of its three dimensions; or it may rotate about any one of its three dimensions as an axis. A single bolt, passing through both beams, as shown in the figure, will prevent all of these movements except rotation about the bolt as an axis. Two

or more bolts will prevent this latter, and consequently, all movement of either of the beams upon the other. A bolt, it may be necessary to say, is a rod of iron whose length is a little greater than the aggregate thickness of the pieces which it fastens together. It is provided at one end with a solid head, and at the other, with a few screw threads on which turns a "nut," for the purpose of gradually compressing together the pieces through which the bolt passes.

96. *Graphical Construction.* Assuming for simplicity's sake in this and in most of these examples, that the timbers are a foot square, and having the given scale; the diagrams will generally explain themselves sufficiently. The scales are expressed fractionally, adjacent to the numbers of the diagrams. The nut only is shown in the plan of this figure.

It is an error to suppose that the nuts and other small parts can be carelessly drawn, as by hand, without injury to the drawing, since these parts easily catch the eye, and if distorted, or roughly drawn, appear very badly.

The method is, therefore, here fully given for drawing a nut accurately. Take any point in the centre line, *ab*, of the bolt, produced, and through it draw any two lines, *cd* and *en*, at right-angles to each other. From the centre, lay off *each way* on *each line*, half the length of each side of the nut, say $\frac{3}{4}$ of an inch.

Then, through the points so found, draw lines parallel to the centre lines *cd* and *en*, and they will form a square plan of a nut $1\frac{1}{2}''$ on each side.

In making this construction, the distances should be set off very carefully, and the sides of the nut *ruled*, in very fine lines, and exactly through the points located. From the plan, the elevation is found as in Pl. II., Fig. 21.

97. Ex. 2. **A Compound Beam, notched and bolted.** Pl. VII., Fig. 47. *Mechanical Construction.* The beams represented in this figure, are indented together by being alternately notched; the portions cut out of either beam being a foot apart, a foot in length, and two inches deep. When merely laid, one upon another, they will offer resistance only to being separated longitudinally, and to horizontal rotation.

The addition of a bolt renders the "compound beam," thus formed, capable of resisting forces tending to separate it in all ways.

Thin pieces are represented, in this figure, between the bolt-head and nut, and the wood. These are circular, having a rounded

edge, and a circular aperture in the middle through which the bolt passes. They are called "*washers*," and their use is, to distribute the pressure of the nut or bolt-head over a larger surface, so as not to indent the wood, and so as to prevent a gouging of the wood in tightening the nut, which gouging would facilitate the decay of the wood, and consequently, the loosening of the nut.

98. *Graphical Construction.*—The beams being understood to be originally one foot square, the compound beam will be 22 inches deep; hence draw the upper and lower edges 22 inches apart, and from each of them, set off, on a vertical line, 10 inches. Through the points, a and b, so found, draw *very faint* horizontal lines, and on either of them, lay off any number of spaces; each, one foot in length. Through the points, as c, thus located, draw transverse lines between the faint lines, and then, to prevent mistakes in inking, make slightly heavier the notched line which forms the real joint between the timbers.

The use of the scale of $\frac{1}{24}$ continues till a new one is mentioned.

The following empirical rules will answer for determining the sizes of nuts and washers on assumed sketches like those of Pl. VII., so as to secure a good appearance to the diagram. The side of the nut may be double the diameter of the bolt, and the greater diameter of the washer may be equal to the diagonal of the nut, plus twice the thickness of the washer itself.

Execution.—This is manifest in this case, and in most of the following examples, from an inspection of the figures.

99. Ex. 3. **A Compound Beam, keyed.** Pl. VII., Fig. 48. *Mechanical Construction.* The defect in the last construction is, that the bearing surfaces opposed to separation in the direction of the length of the beam, present only the ends of the grain to each other. These surfaces are therefore liable to be readily abraded or made spongy by the tendency to an interlacing action of the fibres. Hence it is better to adopt the construction given in Pl. VII., Fig. 48, where the "*keys*," as K, are supposed to be of hard wood, whose grain runs in the direction of the width of the beam. In this case, the bolts are passed through the keys, to prevent them from slipping out, though less boring would be required if they were placed midway between the keys. In this example, the strength of the beam is greatly increased with but a very small increase of material, as is proved in mechanics and confirmed by experiment.

100. *Graphical Construction.*—This example differs from the last so slightly as to render a particular explanation unnecessary. The

keys are 12 inches in height, and 6 inches in width, and are 18 inches apart from centre to centre. They are most accurately located by their vertical centre lines, as AA′. If located thus, and from the horizontal centre line BB′, they can be completely drawn before drawing ee' and nn'. The latter lines, being then pencilled, only between the keys, mistakes in inking will be avoided.

Execution.—The keys present the end of their grain to view hence are inked in diagonal shade lines, which, in order to render the illuminated edges of the keys more distinct, might terminate, uniformly, at a short distance from the upper and left hand edges.

By shading only that portion of the right hand edge of each key, which is between the timbers, it is shown that the keys do not project beyond the front faces of the timbers.

101. Ex. 4. **A Compound Beam, scarfed.** Pl. VII., Fig. 49. *Mechanical Construction.* This specimen shows the use of a *series of shallow notches* in giving one beam a firm hold, so to speak, upon another; as *one deep notch*, having a bearing surface equal to that of the four shown in the figure, would so far cut away the lower beam as to render it nearly useless.

102. *Graphical Construction.*—The notches, *one foot* long, and *two inches* deep, are laid down in a manner similar to that described under Ex. 2.

103. *Execution.*—The keys, since they present the end of the grain to view, are shaded as in the last figure. Heavy lines on their right hand and lower edges would indicate that they projected beyond the beam.

Remark.—When the surfaces of two or more timbers lie in the same plane, as in many of these examples, they are said to be "*flush*" with each other.

§ III.—*Combinations of Timbers, whose axes make angles of* 90° *with each other.*

104. The usual way of fastening timbers thus situated, is by means of a projecting piece on one of them, called a "*tenon*," which is inserted into a corresponding cavity in the other, called a "*mortise*." The tenon may have three, two, or one of its sides flush with the sides of the timber to which it belongs; while the mortise may extend entirely, or only in part, through the timber in which it is made, and may be enclosed by that timber on three or on all sides. [See the examples which follow, in which some of these cases are represented, and from which the rest can be understood.]

When the mortise is surrounded on three or on two sides, particularly in the latter case, the framed pieces are said to be "*halved*" together, more especially in case they are of equal thickness, and have half the thickness of each cut away, as at Pl. VII., Fig. 52.

105. Example 1. **Two examples of a Floor Joist and Sill.** (From a Model.) Pl. VII., Fig. 53. *Mechanical Construction.* A—A′ is one sill, B—B′ another. CC′ is a floor timber framed into both of them. At the left hand end, it is merely "dropped in," with a tenon; at the right hand end, it is framed in, with a tenon and "tusk," *e*. At the right end, therefore, it cannot be *lifted* out, but must be *drawn* out of the mortise. The tusk, *e*, gives as great a thickness to be broken off, at the insertion into the sill, and as much horizontal bearing surface, as if it extended to the full depth of the tenon, *t*, above it, while less of the sill is cut away. Thus, labor and the strength of the sill, are saved.

106. *Graphical Construction.*—1*st.* Draw *ab*. 2*d.* On *ab* construct the elevation of the sills, each $2\frac{1}{2}$ inches by 3 inches. 3*d.* Make the two fragments of floor timber with their upper surfaces flush with the tops of the sills, and 2 inches deep. 4*th.* The mortise in A′, is $\frac{3}{4}$ of an inch in length, by 1 inch in vertical depth. 5*th.* Divide *cd* into four equal parts, of which the tenon and tusk occupy the second and third. The tenon, *t*, is $\frac{3}{4}$ of an inch long, and the tusk, *e*, $\frac{1}{4}$ of an inch long. Let the scale of $\frac{1}{6}$ be used.

107. *Execution.*—The sills, appearing as sections in elevation, are shaded. In all figures like this, dotted lines of construction should be freely used to assist in "reading the drawing," i.e. in comprehending, from the drawing, the construction of the thing represented.

108. Ex. 2. **Example of a "Mortise and Tenon," and of "Halving."** (From a Model.) Pl. VII., Fig. 54. *Mechanical Construction.* In this case, the tenon, AA′, extends entirely through the piece, CC′, into which it is framed. B and C are halved together, by a mortise in each, whose depth equals half the thickness of B, as shown at B″ and C″, and by the dotted line, *ab*.

Graphical Construction.—Make, 1*st*, the elevation, A′; 2*d*, the plan; 3*d*, the details. B″ is an elevation of B as seen when looking in the direction, BA. C″ is an elevation of the left hand portion of CC′, showing the mortise into which B is halved. The dimensions may be assumed, or found by a scale, as noticed below.

109. *Execution.*—The invisible parts of the framing, as the halv

ing, as seen at *ab* in elevation, are shown in dotted lines. The brace and the dotted lines of construction serve to show what separate figures are comprehended under the general number (54) of the diagram. The scale is $\frac{1}{6}$. From this the dimensions of the pieces can be found on a scale.

110. Ex. 3. **A Mortise and Tenon as seen in two sills and a post. Use of broken planes of section.** (From a Model.) Pl. VII., Fig. 55.

Mechanical Construction.—The sills, being liable to be drawn apart, are pinned at *a*. The post, BB′, is kept in its mortise, *bb″*, by its own weight; *m* is the mortise in which a vertical wall joist rests. It is shown again in section near *m′*.

111. *Graphical Construction.*—The plan, two elevations, and a broken section, show all parts fully.

The assemblage is supposed to be cut, as shown in the plan by the broken line AA′A″A‴, and is shown, thus cut, in the shaded figure, A′A′A‴*m′*. The scale, which is the same as in Fig. 53, indicates the measurements. At B″, is the side elevation of the model as seen in looking in the direction A′A.

In Fig. 55 *a*, A′*s* obviously equals A″*s*, as seen in the plan.

112. *Execution.*—In the shaded elevation, Fig. 55*a*, the cross-section, A′A‴, is lined as usual. The longitudinal sections are shaded by longitudinal shade lines. The plan of the broken upper end of the post, B, is filled with arrow heads, as a specimen of a way sometimes convenient, of showing an end view of a broken end.

Sometimes, though it renders the execution more tedious, narrow blank spaces are left on shaded ends, opposite to the heavy lines, so as to indicate more plainly the situation of the illuminated edges (100). The shading to the left of A′, Fig. 55*a*, should be placed so as to distinguish its surface from that to the right of A′.

113. Ex. 4. **A Mortise and Tenon, as seen in timbers so framed that the axis of one shall, when produced, be a diagonal diameter of the other.** Pl. VII., Fig. 56. *Mechanical Construction.*—In this case the end of the inserted timber is not square, and in the receiving timber there is, besides the mortise, a tetraedron cut out of the body of that timber.

114. *Graphical Construction.*—D is the plan, D′ the side elevation, and D″ the end elevation of the piece bearing the tenon. F′ and F are an elevation and plan of the piece containing the mortise. Observe that the middle line of D, and of D′, is an axis of symmetry, and that the oblique right hand edges of D and D′ are parallel to the corresponding sides of the incision in F′.

§ IV.—*Miscellaneous Combinations.*

115. EXAMPLE 1. **Dowelling.** (From a Model.) Pl. VII., Fig. 57. *Mechanical Construction.—Dowelling* is a mode of fastening by pins, projecting usually from an edge of one piece into corresponding cavities in another piece, as seen in the fastening of the parts of the head of a water tight cask. The mode of fastening, however, rather than the relative position of the pieces, gives the name to this mode of union.

The example shown in Pl. VII., Fig. 57, represents the braces of a roof framing as dowelled together with oak pins.

116. *Graphical Construction.*—This figure is, as its dimensions indicate, drawn from a model. The scale is one-third of an inch to an inch.

1st. Draw *acb*, with its edges making any angle with the imaginary ground line—not drawn.

2d. At the middle of this piece, draw the pin or *dowel*, *pp*, $\frac{1}{4}$ of an inch in diameter, and projecting $\frac{3}{4}$ of an inch on each side of the piece, *acb*. This pin hides another, supposed to be behind it.

3d. The pieces, *d* and *d''*, are each $2\frac{1}{2}$ inches by 1 inch, and are shown as if just drawn off from the *dowels*, but in their true direction, i.e. at right angles to *acb*.

4th. The inner end of *d* is shown at *d'*, showing the two holes, $1\frac{1}{2}$ inches apart, into which the *dowels* fit.

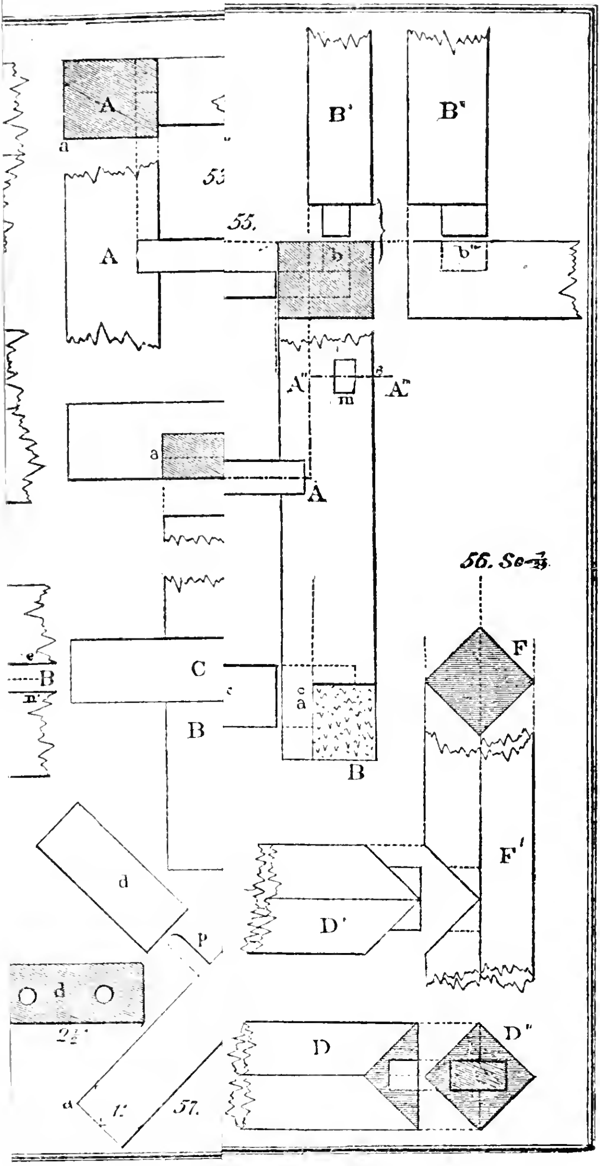
Execution.—The end view is lined as usual, leaving the dowel holes blank.

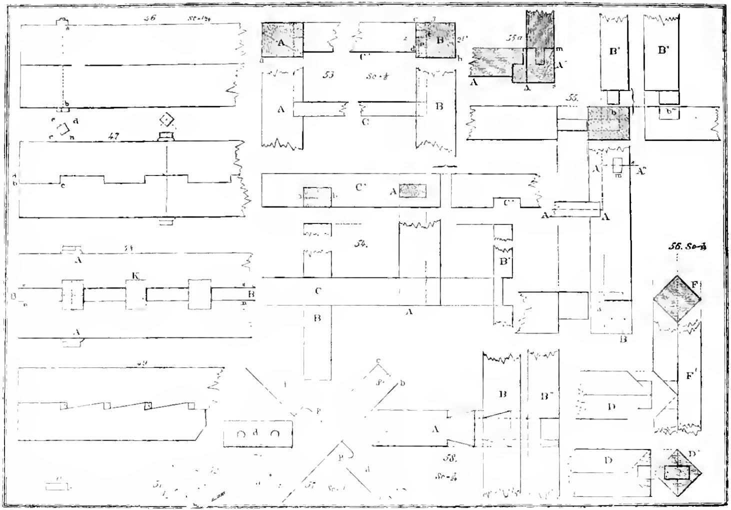
117. Ex. 2. **A dovetailed Mortise and Tenon.** Pl. VII., Fig. 58. *Mechanical Construction.*—This figure shows a species of joining called *dovetailing*. Here the mortise increases in width as it becomes deeper, so that pieces which are dovetailed together, either at right angles or endwise, cannot be pulled directly apart. The corners of drawers, for instance, are usually dovetailed; and sometimes even stone structures, as lighthouses, which are exposed to furious storms, have their parts dovetailed together.

118. *Graphical Construction.*—The sketches of this framing are arranged as two elevations. A bears the dovetail, B shows the length and breadth of the mortise, and B'' its depth. A and B belong to the same elevation.

Execution.—In this case a method is given, of representing a hidden cut surface, viz. by dotted shade lines, as seen in the hidden faces of the mortise in B''.

119. Leaving now the examples of pieces framed together at right angles let us consider:—


A
a
53
A
55.
B'
B'
b
b'''
A''
A''
m
a
A
e
B
n
C
B
a
B
56. Sc
F
F'
d
p
D'
d
2½
D
D''
a
57.



§ V.—*Pairs of Timbers which are framed together obliquely to each other.*

EXAMPLE 1. **A Chord and Principal.** (From a Model.) Pl. VIII., Fig. 59. *Mechanical Construction.*—The oblique piece ("principal") is, as the two elevations together show, of equa width with the horizontal piece ("chord," or "tie beam"), and i framed into it so as to prevent sliding sidewise or lengthwise.

Neither can it be lifted out, on account of the bolt which is made to pass perpendicularly to the joint, *ac*, and is "chipped up" at *pp*, so as to give a flat bearing, parallel to *ac*, for the nut and bolt-head.

120. *Graphical Construction.*—1*st.* Draw *pde*; 2*d.* Lay off *de* $=13$ inches; 3*d.* Make $e'ea=30°$; 4*th.* At any point, e', draw a perpendicular to ee', and lay off upon it 9 inches—the perpendicular width of $e'ea$; 5*th.* Make $ec=4$ inches and perpendicular to $e'e$; bisect it and complete the outlines of the tenons, and the shoulder acc'; 6*th.* To draw the nut accurately, proceed as in Pl. VII., Fig. 46–47, placing the centre of the auxiliary projection of the nut in the axis of the bolt produced, &c. (46) (96). *b* represents the bolt hole, the bolt being shown only on one elevation.

121. Ex. 2. **A Brace, as seen in the angle between a "post" and "girth."** (From a Model.) Pl. VIII., Fig. 60. *Mechanical Construction.*—PP′ is the post, GG′ is the girth, and B′B″ is the brace, having a truncated tenon at each end, which rests in a mortise. When the brace is quite small, it has a shoulder on one side only of the tenon, as if B′B″ were sawed lengthwise on a line, oo'.

122. *Graphical Construction.*—To show a tenon of the brace clearly, the girth and brace together are represented as being drawn out of the post. 1*st.* Draw the post. 2*d.* Half an inch below the top of the post, draw the girth $2\frac{1}{2}$ inches deep. 3*d.* From *a*, lay off *ab* and *a* each 4 inches, and draw the brace 1 inch wide. 4*th.* Make *cd* equal to the adjacent mortise; viz. $1\frac{1}{2}$ inches; make $de=1$ inch, and erect the perpendicular at *e* till it meets *bc*, &c. The dotted projecting lines show the construction of B″ and of the plan. At e'' is the vertical end of the tenon *e*. On each side of e'', are the vertical surfaces, shown also at *cd*. Let B′ also be projected on a plane parallel to *bc*.

123. Ex. 3. **A Brace, with shoulders mortised into the post.** Pl. VIII., Fig. 61. This is the strongest way of framing a brace. For the rest, the figure explains itself. Observe, however, that while in Fig. 59, the head of the tenon and shoulder is perpendi-

cular to the oblique piece, here, where that piece is framed into a vertical post, its head *nu* is perpendicular to the axis of the post. In Fig. 61, moreover, the auxiliary plane on which the brace alone is projected, is parallel to the length of the brace, as is shown by the situation of B, the auxiliary projection of the brace, and by the direction of the projecting lines, as *rr'*. P' is the elevation of P, as seen in the direction *nn'*, and with the brace removed.

124. Ex. 4. **A "Shoar."** Pl. VIII., Fig. 62. *Mechanical Construction.*—A "shoar" is a large timber used to prop up earth or buildings, by being framed obliquely into a horizontal beam and a stout vertical post. It is usually of temporary use, during the construction of permanent works; and as respects its action, it resists compression in the direction of its length. To give a large bearing surface without cutting too far into the vertical timber, it often has two shoulders. The surface at *ab* is made vertical, for then the fibres of the post are unbroken except at *cb*, while if the upper shoulder were shaped as at *adb*, the fibres of the triangular portion *dbc* would be short, and less able to resist a longitudinal force.

125. The *Graphical construction* is evident from the figure, which is in two elevations, the left hand one showing the post only.

Execution.—The vertical surface at *ab* may, in the left hand elevation, be left blank, or shaded with vertical lines as in Pl. VII., Fig. 55*a*.

§ VI.—*Combinations of Timbers whose axes make angles of* 180° *with each other.*

126. Timbers thus framed are, in general, said to be spliced. Six forms of splicing are shown in the following figures.

Example 1. **A Halved Splicing, pinned.** Pl. VIII., Fig. 63. The *mechanical construction* is evident from the figure. When boards are lapped on their edges in this way, as in figure 69, they are said to be "*rabbetted.*"

127. *Graphical Construction.*—After drawing the lines, 12 inches apart, which represent the edges of the timber, drop a perpendicular of 6 inches in length from any point as *a*. From its lower extremity, draw a horizontal line 12 inches in length, and from *c*, drop the perpendicular *cb*, which completes the elevation. In the plan, the joint at *a* will be seen as a full line *a'a"*, and that at *b*, being hidden is represented as a dotted line, at *b"*

Draw a diagonal, $a'b''$, divide it into four equal parts, and take the first and third points of division for the centres of two pins, having each a radius of three-fourths of an inch.

Execution.—The position of the heavy lines on these figures is too obvious to need remark.

128. Ex. 2. **Tonguing and Grooving; and Mortise and Tenon Splicing.** Pl. VIII., Fig. 64. Boards united at their edges in this way, as shown in Pl. VIII., Fig. 70, are said to be tongued and grooved.

Drawing, as before, the plan and elevation of a beam, a foot square, divide its depth *an*, at any point *a*, into five equal parts. Take the second and fourth of these parts as the width of the tenons, which are each a foot long.

The joint at *a* is visible in the plan, the one at *b* is not. Let $a'd$ be a diagonal line of the square $a'd$. Divide $a'd$ into three equal parts, and take the points of division as centres of inch bolts, with heads and nuts 2 inches square, and washers of $1\frac{3}{4}$ inches radius. To place the nut in any position on its axis, draw any two lines at right angles to each other, through each of the bolt centres, and on each, lay off 1 inch from those centres, and describe the nut. Project up those angles of the nut which are seen; viz. the foremost ones, make it 1 inch thick in elevation, and its washer $\frac{1}{2}$ an inch thick.

In this, and all similar cases, the head of the bolt, *s*, would not have its longer edges necessarily parallel to those of the nut. To give the bolt head any position on its axis, describe it in an auxiliary plan just below it.

129. Ex. 3. **A Scarfed Splicing, strapped.** Pl. VIII., Fig. 65. *Mechanical Construction.*—While timbers, framed together as in the two preceding examples, can be directly slid apart when their connecting bolts are removed; the timbers, framed as in the present example, cannot be thus separated longitudinally, on account of the dovetailed form of the splice. Strapping makes a firm connexion, but consumes a great deal of the uniting material.

130. *Graphical Construction.*—After drawing the outlines of the side elevation, make the perpendiculars at *a* and a', each 8 inches long, and make them 18 inches apart. One inch from *a*, make the strap, ss', $2\frac{1}{2}$ inches wide, and projecting half an inch—i. e. its thickness—over the edges of the timber. The ear through which the bolt passes to bind the strap round the timber, projects two inches above the strap. Take the centre of the ear as the centre of the bolt, and on this centre describe the bolt head $1\frac{1}{2}$ inches square.

In the plan, the ears of the strap are at any indefinite distance apart, depending on the tightness of the nut, *s*.

A fragment of a similar strap at the other end of the scarf, is shown, with its visible ends on the bottom, near *a'*, and back, at *r* of the beam.

Execution.—The scarf is dotted where it disappears behind the strap; and so are the hidden joint at *a'*, and the fragment of the second strap, as shown in plan. These examples may advantageously be drawn by the student on a scale of $\frac{1}{12}$. Care must then be exercised in making the large broken ends neatly, in large splinters, edged with fine ones.

131. Ex. 4. **A Scarfed Splice, bolted.** Pl. VIII., Fig. 66. *Mechanical Construction.*—AA' is one timber. BB' is the other. *Each* is cut off as at *aea''*, forming a pointed end which prevents lateral displacement. *kf* and *dg* are the ends of transverse keys, which afford good bearing surfaces. See the same on Pl. XIII., Fig. 105, which shows the arrangement plainly.

132. *Graphical Constrvction.*—Let each timber be 1 foot square and let the scale be 1 foot to 1 inch. Draw the outlines accordingly, and, assuming *a'*, make $a'b'=3$ feet; drop the perpendicular *b'c'*, and draw a line of construction *a'c'*. From *a'* lay off 4 inches on *a'c'* and divide the rest of *a'c'* into three equal parts, to get the size of the *equal* spaces *c'k*, *fd* and *gv'*, and at *c'* and *k* draw perpendiculars to *a'c'*, *above* it and 2 inches long. Divide *a'k* into two equal parts at *d*, and from *k* and *d* lay off 2 inches on *a'c'* towards *a'* and complete the keys, as shown, *kf* above *a'c'*, and *dg* below it; also the joints *v'g*, *dk*, etc., of the splice, where *a'v'* is perpendicular to *a'c'* and 2 inches long.

Now, in plan, project down *a'* at *a* and *a''* and draw *ae* and *a''e* at 60° with *a''c*, and do the same with *c'*, as shown. Project up *e* to *e'* and draw *e's'* parallel to *a'v'* till it meets *c'g* produced. Then project down *v'* at *v* and *v''*, and *s'* at *s*, and draw *sv* and *sv''*, which, of course, will not be parallel to *ae* and *a''e*, since *se*—*s'e'* is shorter than *av*—*a'v'*. Thus *av se a''v''*—*a'v' s'e'* is the obliquely pointed end of the piece AA'. BB' is similarly pointed, as shown.

Add the bolts, nuts and washers, *nn'* and *mm'*, in any convenient position, which will complete the construction.

Execution.—The keys are shaded. The hidden cut surfaces of the notches are shaded in dotted shade lines, and the hidden joints are dotted.

133. Ex. 5. **A Compound Beam, with one of the component beams "fished."** Pl. VIII., Fig. 67. *Mechanical Construc*

tion.—The mode of union called *fishing*, consists in uniting two pieces, end to end, by laying a notched piece over the joint and bolting it through the longer pieces.

The figure shows this mode as applied to a compound beam, i. e. to a beam "*built*" of several pieces bolted and keyed together. The order of construction is as follows—taking for a scale $\frac{3}{8}$ of an inch to a foot.

134. *Graphical Construction.*—1*st.* The outside lines of the plan are two feet apart, the outside pieces are each $4\frac{1}{2}$ inches wide, and the interior ones $5\frac{1}{2}$ inches; leaving four inches for the sum of the three equal spaces between the four beams.

2*d.* Let there be a joint at a'. Lay off 3 feet, each side of a' for the length of the "*fish.*"

3*d.* The straight side of this piece is let into the whole piece, b, two thirds of an inch, and into a', 1 inch, making its thickness 3 inches.

4*th.* At a', it is two inches thick, i. e. at a', the timber, a', is of its full thickness. The fish, c, is 2 inches thick for the space of one foot at each side of a'. The notches at d and e are each 1 inch deep, dd' and ee' each are one foot, and the notches at d' and e are each 1 inch deep. The remaining portions of the fish are 1 foot long, and 3 inches wide.

5*th.* Opposite to these extreme portions, are keys, 1 foot by 3 inches, in the spaces between the other timbers, and setting an equal depth into each timber.

6*th.* In elevation, only the timber a' is seen—1 foot deep. Four bolts pass through the keys. $b'b''=5$ feet, and b' is three inches from the top, and from kk', the left hand end of the fish. $n'n''=5$ feet, and n' is 3 inches above the bottom of the timber, and 9 inches from kk'. The circular bolt head is one inch in diameter, and its washer $3\frac{1}{2}$ inches diameter and $\frac{1}{2}$ an inch thick. The thickness of the bolt head, as seen in plan, is $\frac{3}{4}$ inch. The nuts, nn, are $1\frac{1}{2}$ inches square, and $\frac{1}{2}$ an inch thick, and the bolts are half an inch in diameter.

135. The several nuts would naturally be found in various positions on their axes. To construct them thus with accuracy, as seen in the plan, one auxiliary elevation, as N, is sufficient. N, and its centre, may be projected upon as many planes—xy—as there are different positions to be represented in the plan, each plane being supposed to be situated, in reference to N, as some nut in the plan is, to its elevation. Then transfer the points on xy, &c., to the outside of the several *nut*-washers, placing the projection of the centre lines of the bolts in the plan, as lines of reference.

Execution.—The figure explains itself in this respect.

136. Ex. 6. **A vertical Splice.** Pl. VIII., Fig. 68. *Mechanical Construction.*—This splice is formed of two prongs at opposite corners of each piece, embraced by corresponding notches in the other piece. Thus in the piece B′, the visible prong, as seen in elevation, is a truncated triangular pyramid whose horizontal base is *abc—a′c′*, and whose oblique base is *enc—e′n″c″*. Besides the four prongs, two on each timber, there is a flat surface *abfq—c′a′q′*, well adapted to receive a vertical pressure, since it is equal upon, and common to, both timbers.

137. *Graphical Construction.*—To aid in understanding this combination, an oblique projection is given on a diagonal plane, parallel to PQ.

1*st.* Make the plan, *acfq*, with the angles of the interior square in the middle of the sides of the outer one. 2*d.* Make the distances, as $ce=2$ inches and draw *en*, &c. 3*d.* Make $c'n'=c'n''$ 15 inches, and draw short horizontal lines, *n″e′*, &c., on which project *e*, &c., after which the rest is readily completed.

138. *Execution.*—Observe, in the plans, to change the direction of the shade lines at every change in the position of the surface of the wood.

CHAPTER III.

CONSTRUCTIONS IN METAL.

139. EXAMPLE 1. **An end view of a Railroad Rail.** Pl. VIII., Fig. 71. *Graphical Construction.*—1*st.* Draw a vertical centre line AA′, and make AA′$=3\frac{3}{4}$ inches. 2*d.* Make A′b=A′b'=2 inches. 3*d.* Make Ac=Ac'=1 inch. 4*th.* Describe two quadrants, of which $c'd$ is one, with a radius of half an inch. 5*th.* With p, half an inch from AA′, as a centre, and pd as a radius, describe an arc, dr, till it meets a vertical line through e. 6*th.* Draw the tangent rs. 7*th.* Draw a vertical line, as pt', $\frac{1}{2}$ an inch from AA′, on each side of AA′. 8*th.* Bisect the angle rst' and note s', where the bisecting line meets the radius, pr, produced. 9*th.* With s' as a centre, draw the arc rt'. 10*th.* At b and b' erect perpendiculars, each one fourth of an inch high. 11*th.* Draw quadrants, as $q't$, tangent to these perpendiculars and of one fourth of an inch radius. 12*th.* Draw the horizontal line tv. 13*th.* Make $nt'=nv$ and describe the arc $t'v$. 14*th.* Repeat these operations on the other side of the centre line, AA′.

Execution.—Let the construction be fully shown on one side of the centre line.

140. Ex. 2. **An end elevation of a Compound Rail.** Pl. VIII., Fig. 72. *Mechanical Construction.*—The compound rail, is a rail formed in two parts, which are placed side by side so as to break joints, and then riveted together. As one half of the rail is whole, at the points where a joint occurs on the other half, the noise and jar, observable in riding on tracks built in the ordinary manner, are both obviated; also "chairs," the metal supports which receive the ends of the ordinary rails, may be dispensed with, in case of the use of the compound rail.

In laying a compound rail on a curve, the holes, through which the bolts pass, may be drawn past one another by the bending of the rails. To allow for this, these holes are "slotted," as it is termed, i. e. made longer in the direction of the length of the rail.

141. *Graphical Construction.*—1*st.* Make $ty=4$ inches. 2*d.* Bisect ty at u, and erect a perpendicular, ua, of $3\frac{1}{2}$ inches. 3*d.*

Make, successively, $ur = \frac{1}{2}$ an inch; from r to $cb = 2$ inches; from n to $nh = \frac{7}{8}$ of an inch; and to $ge = 2\frac{1}{8}$ inches. *4th.* For the several widths of the interior parts, make $bc = \frac{3}{8}$ of an inch, and g and e each $\frac{3}{8}$ of an inch, from ua; $nh = ge$, and $ro = \frac{3}{8}$ of an inch. *5th.* To locate the outlines of the rail, make ms, the flat top, called the tread of the rail, $= 2$ inches, half an inch below this, make the width, td, 3 inches; and make the part through which the rivet passes, $1\frac{3}{4}$ inches thick, and rounded into the lower flange which is $\frac{3}{4}$ of an inch thick.

The rivet has its axis $1\frac{5}{8}$ inches from ty. Its original head, q, is conical, with bases of—say 1 inch, and $\frac{3}{4}$ of an inch, diameter; and is half an inch thick. The other head, p, is made at pleasure, being roughly hammered down while the rivet is hot, during the process of track-laying. A thin washer is shown under this head.

142. Ex. 3. **A "Cage Valve," from a Locomotive Pump.** Pl. VIII., Fig. 73–74.—*Mechanical Construction.*—This valve is made in three pieces, viz. the valve proper, Fig. 74; the cage containing it, Abb'; and the flange $bb'c$; whose cylindrical aperture—shown in dotted lines—being smaller than the valve, confines it. The valve is a cup, solid at the bottom, and makes a water tight joint with the upper surface of the flange, inside of the cage. The whole is inclosed in a chamber communicating with the pump barrel, and with the tender, or the boiler, according as we suppose it to be the inlet or outlet valve of the pump. This chamber makes a water tight joint with the circumference of the flange bb'.

Suppose the valve to be the latter of the two just named. The "plunger" of the pump being forced in, the water shuts the inlet valve, and raises the outlet valve, and escapes between the bars of the cage into the chamber, and from that, by a pipe, into the boiler.

143. *Graphical Construction.*—Scale full size. Make the plan first, where the six bars are equal and equidistant, with radial sides. Project them into the elevation; as in Prob. 14, Div. I.; taking care to note whether any of the bars, as E, on the back part of the cage can be seen above the valve, CD, and between the front bars, as F and G. The diameters of the circles seen in the *cage* are, in order, from the centre, $1\frac{1}{4}$, $2\frac{1}{8}$, $2\frac{15}{16}$, and 4 inches. The thickness of the valve, Fig. 74, is $\frac{3}{16}$ of an inch, its outside height $1\frac{3}{8}$ inches, and the outside diameter $2\frac{3}{16}$ inches. The diameter of the aperture in the flange is $1\frac{3}{4}$ inches, its length $\frac{3}{4}$ of an inch, and the height of the whole cage is $3\frac{3}{16}$ inches.

144. *Execution.*—Observe carefully the position of the heavy lines. The section, Fig. 74, being of metal, is finely shaded.

145. Ex. 4. **An oblique elevation of a Bolt Head.** Pl. VIII., Fig. 75. Let PQ be the intersection of two vertical planes, at right angles with each other; and let RS be the intersection, with the vertical plane of the paper to the left of PQ, of a plane which, in space, is parallel to the square top of the bolt head. On such a plane, a plan view of the bolt head may be made, showing two of its dimensions in their real size; and on the plane above RS, the thickness of the bolt head, and diameter of the bolt, are shown in their real size.

Below RS, construct the plan of the bolt head, with its sides making any angle with the ground line RS. Project its corners in perpendiculars to RS, giving the left hand elevation, whose thickness is assumed.

146. The fact that the projecting lines of a point, form, in the drawings, a perpendicular to the ground line, is but a special case of a more general truth, which may be thus stated.—When an object in space is projected upon any two planes which are at right angles to each other, the projecting lines of any point of that object form a line, in the drawing, perpendicular to the intersection of the two planes.

147. To apply the foregoing principle to the present problem; it appears that each point, as a'', of the right hand elevation, will be in a line, $a'a''$, perpendicular to PQ, the intersection of the two vertical planes of projection.

Remembering that PQ is the intersection of a vertical plane—perpendicular to the plane of the paper—with the vertical plane of the paper, and observing that the figure represents this plane as being revolved around PQ towards the left, and into the plane of the paper, and observing the arrow, which indicates the direction in which the bolt head is viewed, it appears that the revolved vertical plane, has been transferred from a position at the left of the plan *acne*, to the position, PQ, and that the centre line tu'', must appear as far from PQ as it is in front of the plane of the paper—i. e. $e''u''=eu$, showing also, that as e—e' is in the plane of the paper, its projection at e'' must be in PQ, the intersection of the two vertical planes.

Similarly, the other corners of the nut, as c'', n'', &c., are laid off either from the centre line tu'', or from PQ. Thus $v''c''=vc$, or $v'''c''=sc$. The diameter of the bolt is equal in both elevations.

148. Other supposed positions of the auxiliary plane PQ may be assumed by the student, and the corresponding construction worked out. Thus, the primitive position of PQ may be at the right of the

bolt head, and that may be viewed in the opposite direction from that indicated by the arrow.

149. Ex. 5. **A "Step" for the support of an oblique timber.** Pl. VIII., Fig. 76. *Mechanical Construction.*—It will be frequently observed, in the framings of bridges, that there are certain timbers whose edges have an oblique direction in a vertical plane, while at their ends they abut against horizontal timbers, not directly, for that would cause them to be cut off obliquely, but through the medium of a prismatic block of wood or iron, so shaped that one of its faces, as ab—$n'b'$, Fig. 76, rests on the horizontal timber, while another, as ac—$e''d''c''$, is perpendicular to the oblique timber.

To secure lightness with strength, the step is hollow underneath, and strengthened by ribs, rr. The holes, $h'h''$, allow the passage of iron rods, used in binding together the parts of the bridge. These holes are here prolonged, as at h, forming tubes, which extend partly or wholly through the horizontal timber on which the step rests, in order to hold the step steadily in its place.

150. *Remark.* When the oblique timber, as T, Fig. 76 (a), sets *into*, rather than *upon*, its iron support S, so that the dotted lines, ab and cb, represent the ends of the timber, the support, S, is called a shoe.

151. *Graphical Construction.*—In the plate, abc is the elevation, and according to the usual arrangement would be placed above the plan, $e''n''c''$, of the top of the step. $a'b'e$ is the plan of the under side of the step, showing the ribs, &c. A line through nm is a centre line for this plan and for the elevation. A line through the middle point, r, of mn, is another centre line for the plan of the bottom of the step. Having chosen a scale, the position of the centre lines, and the arrangement of the figures, the details of the construction may be left to the student.

152. Ex. 6. **A metallic steam tight "Packing," for the "stuffing boxes" of piston rods.** Pl. VIII., Fig. 77.

Mechanical Construction.—Attached to that end, b, of Fig. 77 (a), of a steam cylinder, for instance, at which the piston rod, p, enters it, is a cylindrical projection or "neck," n, having at its outer end a flange, ff, through which two or more bolts pass. At its inner end, at p, this neck fits the piston rod quite close for a short space. The internal diameter of the remaining portion of the neck is sufficient to receive a ring, rr, which fits the piston rod, and has on its outer edge a flange, t, by which it is fastened to the flange, ff, on the neck of the cylinder by screw bolts. The remaining hollow

space, s, between the ring or "gland," tr, and the inner end of the neck, is usually filled with some elastic substance, as picked hemp, which, as held in place by the gland, tr, makes a steam tight joint; which, altogether, is called the "stuffing box."

153. The objection to this kind of packing is, that it requires so frequent renewals, that much time is consumed, for instance in rail road repair shops, in the preparation and adjustment of the packing. To obviate this loss of time, and perhaps because it seems more neat and trim to have all parts of an engine metallic, this metallic packing, Fig. 77, was invented. ABC—A'B' is a cast iron ring, cylindrical on the outside, and having inlaid, in its circumference, bands, tt', of soft metal, so that it may be squeezed perfectly tight into the neck of the cylinder. The inner surface of this ring is conical, and contains the packing of block tin. This packing, as a whole, is also a ring, whose exterior is conical, and fits the inner side of the iron ring, and whose interior, fkc, is cylindrical, fitting the piston rod closely.

154. For adjustment, this tin packing is cut horizontally into three rings, and each partial ring is then cut vertically, as shown in the figure, into two equal segments. $abcd$—$a'b'c'd'$, is one segment of the upper ring; $efgh$—$e'f'f''g'h'$, is one segment of the middle ring, and $rjkl$—$r'j'k'n'l$, is one segment of the lower ring. The segments of each ring, it therefore appears, break joints with each of the other rings. Three of the segments, one in each partial ring, are loose, while the other three are dowelled by small iron pins, parallel to the axis of the whole packing.

155. *Operation.*—Suppose the interior, fkc, of the packing to be of less diameter than the piston rod, which it is to surround. By drawing it partly out of its conical iron case, the segments forming each ring can be slightly separated, making spaces at ab, &c., which will increase the internal diameter, so as to receive the piston rod. When in this position, let the gland, tr, be brought to bear on the packing, and it will be firmly held in place; then, as the packing gradually wears away, the gland, by being pressed further into the neck, will press the packing further into its conical seat, which will close up the segments round the piston rod.

156. *Graphical Construction.*—Let the scale be from one half to the whole original size of the packing for a locomotive valve chest. C is the centre for the various circles of the plan, and DD', projected up from C, is a centre line for portions of the elevation $Cf = \frac{9}{16}$ of an inch; $Ce = 1\frac{3}{16}$ inches; $CA = 1\frac{1}{4}$ inches. $A'n = 2$ inches, and $A's' = \frac{9}{16}$ of an inch. The iron case being constructed

from these measurements, the rings must be located so that $f'p'$, for instance, shall be $= \frac{7}{16}$ of an inch; and then let the thickness, $f'f''$, of each segment be $\frac{11}{16}$ of an inch. These dimensions and the consequent arrangements of the rings will give spaces between the segments, as at *ab*, of $\frac{1}{8}$ of an inch; though in fact, as this space is variable, there is no necessity for a precise measurement for it. In the plan, there are shown one segment, and a fragment, *abef*, of another, in the upper ring; one segment and two fragments of the middle ring, and both segments of the lower ring, with the whole of the iron case. The elevation shows one of the halves of each ring, viz., $acd—a'c'd'$, the upper one; $ef\,gh—e''f'\,g'h'$, the middle one; and $rjkl—r'j'k'l'$, the lower one.

The circles of the plan are found by projecting the points as p' upon the diameter AC. Then by comparing kk', ll', and nn', for example, we see how the vertical projections, as $k'l'n'$, of the ends of the half rings are found.

157. *Execution.*—The section lines in the elevation indicate clearly the situation of the three segments, *abcd*, *efgh*, and *ijkl*, there shown. The dark bands on the case at t and t', indicate the inlaid bands of soft metal already described.

The student can usefully multiply these examples by constructing *plans, elevations and sections*, from *measurement*, of such objects as steam, water, and gas *cocks*, *valves*, or *gates*, *railway joints*, and *chairs*, and other like simple metallic details; actual examples of which can be easily obtained as models almost anywhere.

Ex. 7. **Rolled Iron Beams and Columns.**

Wood is perishable and growing scarce. *Stone* is comparatively costly and cumbrous. *Cast iron* is suspected as treacherous.

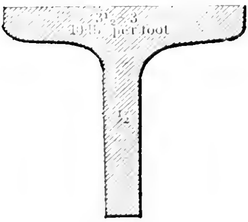


Fig. 1.

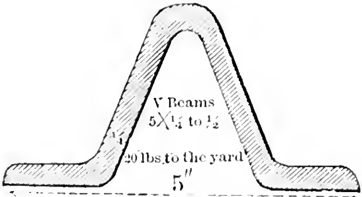
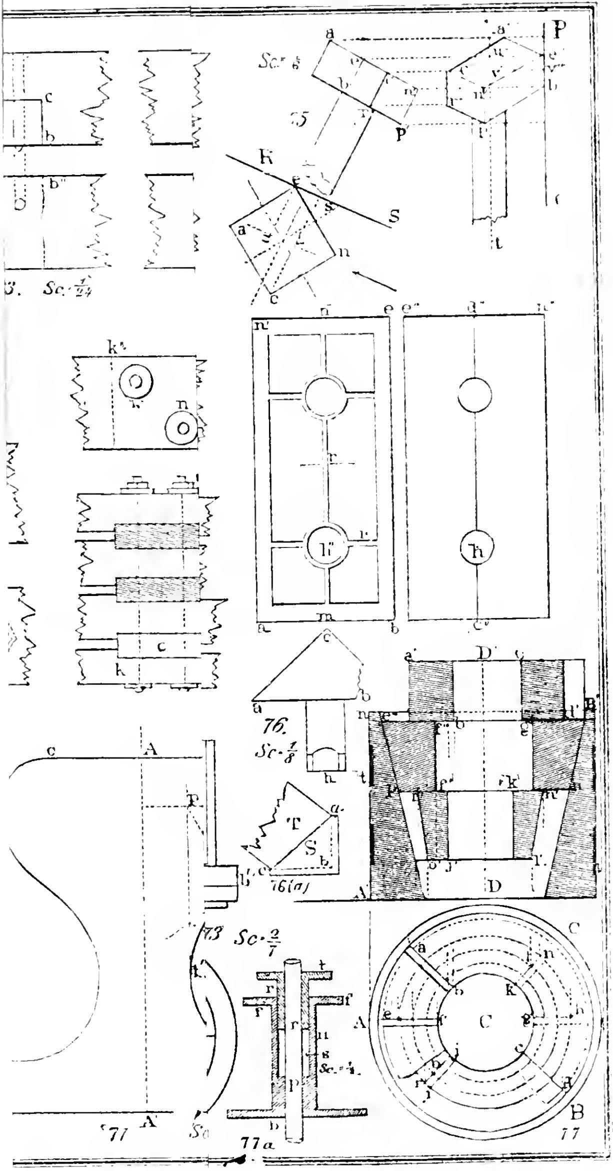
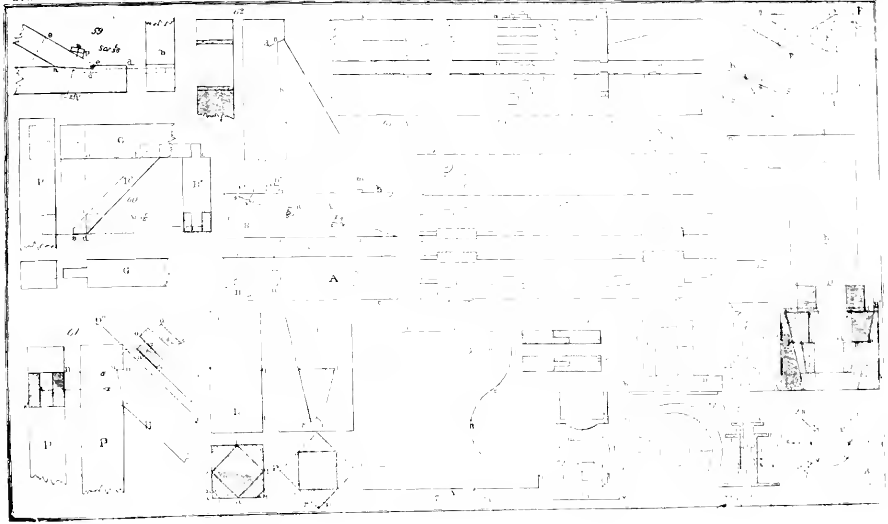


Fig. 2.

Hence, of late years, much attention has been paid to beams and columns of rolled wrought iron. Various figures of such work are therefore given as an appropriate concluding general example for the present chapter.


Sc. 1/24
76.
Sc. 1/8
76(a)
73 Sc. 2/7
77
77a
77


59
Scarfe
60
61
62
G
P
A
F

Rolled iron in its elementary commercial forms, for architectural and engineering purposes, is principally known as *beam, plate, angle*, **T** (Fig. 1), V (Fig. 2), and *channel* iron, the latter

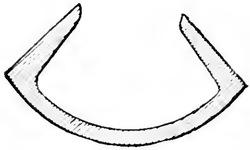


Fig. 3.

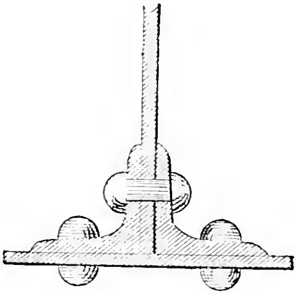


Fig. 4.

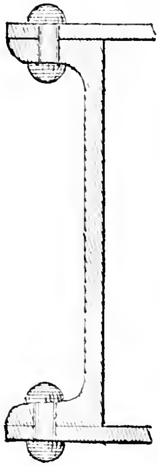


Fig. 5.

either in polygonal or circular segments (Fig. 3). All of these are used in building up compound beams, braces, or columns.

Figs. 4–10, of those here given,* may all be taken as on a scale

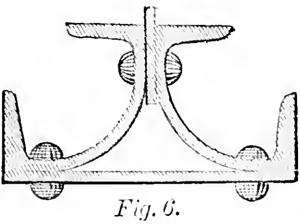


Fig. 6.

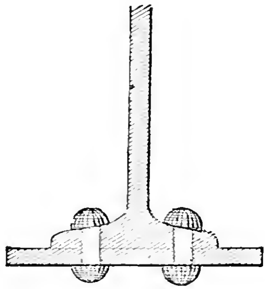


Fig. 7.

of *one fourth* the full size, and Fig. 9, as a practical *example of oblique projection* (see Div. IV.), of *one eighth* the full size.

* From the Union Iron Works, Buffalo, N. Y.

Figs. 4–13, except 9, are all *transverse sections* of the beams or columns which they represent.

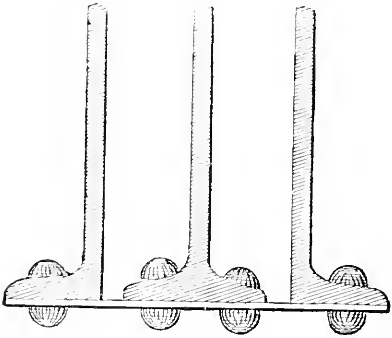


Fig. 8.

Simple beams being made of all sizes from *four* to *fifteen* inches in depth, and some of the sizes either *light* or *heavy*, Figs. 4–8 represent compound beams of depths greater than fifteen inches.

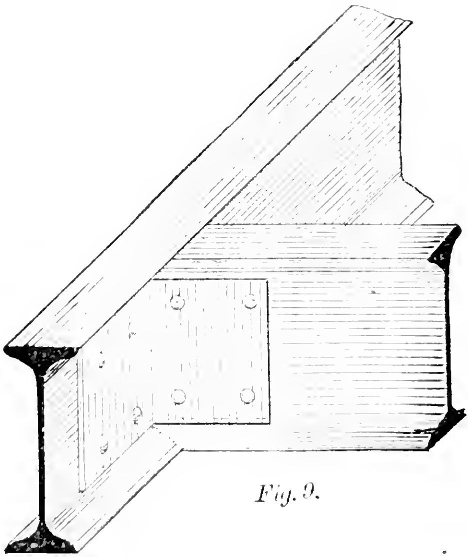


Fig. 9.

Fig. 4 represents the lower half of a beam formed of riveted plate and angle irons.

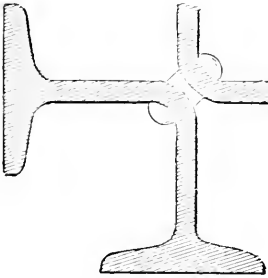


Fig. 10.

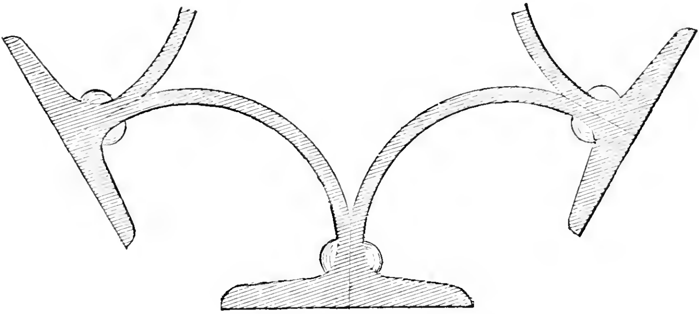


Fig. 11.

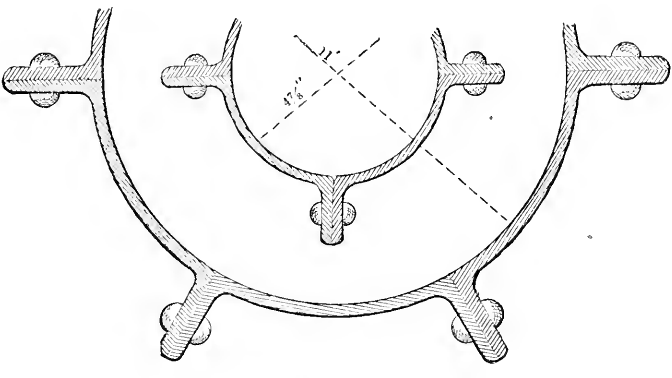


Fig. 12.

Fig. 5 represents the left-hand half of a hollow beam of horizontal plate, and vertical rectangular channel iron.

Fig. 6 shows the lower end of a very deep beam, the ends of which are symmetrically placed, and formed of plate and curved channel iron.

Fig. 7 is the lower half of a beam composed of a simple beam riveted to horizontal plates.

Fig. 8 represents the lower half of a beam, 16″ wide at base, composed of plate, beam, and rectangular channel irons.

Fig. 9, an oblique projection, given here for convenience in anticipation of Div. IV., shows how beams at right angles to each other may be riveted together by angle plates.

Leaving beams for the highly interesting and practically very important subject of iron columns, Fig. 10 is the partial horizontal section of a column curiously formed of two flanged beams, bent at right angles, and riveted along the angle, through an intermediate doubly concave bar, which gives a firmer bearing for the rivets.

Fig. 11 is nearly a half section of a column composed of six curved channel irons, disposed with the convexities inwards, and riveted in the angles of the flanges.

Fig. 12 shows two examples of the true hollow column * as made in segments, consisting of channel irons whose flanges are *radial* and which are placed with their convexities outward, and are riveted through the flanges. The inner figure represents a column of four channel irons, the outer one a larger column of six flanges; the number varying from four to eight in different cases.

Fig. 13 is a section of a column of German design, made of plate and polygonal channel irons.

The ideal of Fig. 12 is a smooth hollow column in one piece, a given amount of matter having much greater strength, in this form, to resist crushing or bending than when in a solid bar. But this ideal, though easily realized in cast iron, is economically impracticable in wrought iron; hence the external radial flanges, though giving additional strength by increasing the average distance of the material from the centre, are subordinate to the main idea.

In Fig. 11, on the contrary, the heavy flanges are the primary feature as a means of gaining strength by a circumferential dis-

* Made at Phœnixville, Pa.

position of material, while the curved parts form a means of connecting them, and of also leaving a hollow interior.

A plate alone easily "buckles" in the direction of its thickness. Each half of Fig. 10 is therefore a plate, braced to prevent this by bending at right angles, and by means of flanges which also make the circumferential strength prominent.

Fig. 13 is also essentially a braced plate column, the primary

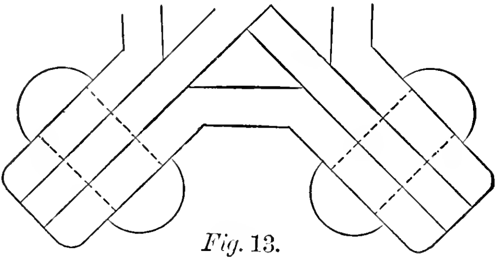


Fig. 13.

office of the channel irons being to unite the plates, leaving their effect on the circumferential strength incidental.

If a column could be formed by placing the channel irons of Fig. 11 so as to join them by their flanges, which, in Fig. 12, would then radiate inward from the circumference of the column, this circumference might then be of slightly greater diameter for a given amount of matter. But the riveting would be inaccessible. It may be a question, however, whether collars might not be clamped or shrunk on, numerous enough to make the column essentially solid, and at no more than the cost of riveting. This question appears to have exercised other minds, and Fig. 14 indicates one solution of it,* consisting of cylindrical segments bearing dovetailed in place of rectangular flanges on their edges, and united by clamping bars in place of rivets; as one would hold two boards laid one upon the other, by clasping each hand over the two edges of the boards, while the riveting plan may be represented by thrusting the fingers through holes near the edges of the boards.

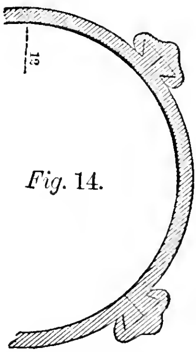


Fig. 14.

* Manufactured by Carnegie Brothers & Co., Pittsburg, Pa.

DIVISION THIRD.

ELEMENTARY SHADOWS AND SHADING.

CHAPTER I.

SHADOWS.

§ I.—*Facts, Principles, and Preliminary Problems.*

158. THE SHADOW of a given opaque body, B, Pl. IX., Fig. 78, upon any surface S, is the portion of S from which light is excluded by B.

A shadow is known when its boundary, *spk*, called the *line of shadow*, is known. Hence, to find the shadow of a given body upon a given surface is, practically, to find the boundary of that shadow.

159. The boundary, *spk*, Pl. IX., Fig. 78, of the shadow of a body, B, is the shadow of the *line of shade, hrn*, which divides the illuminated from the unilluminated part of B. For if a ray, *od*, pierces S, as at *d*, *within* the area of the shadow, it must pierce the body B in its illuminated part as at *o*; but if another ray pierces S, as at *q*, without the shadow, it cannot meet the body B. Hence rays, as *hs*, *cp*, *nk*, which meet S in the *line of shadow*, must be tangent to B at points, as *h*, *c*, *n*, of its *line of shade*.

160. Since the *line of shadow* of any body B on any surface S is the shadow of the *line of shade* of B, the line of shade on the body casting a required shadow must, in general, be found first in problems of shadows.

On plane-sided, that is flat-sided bodies, this line of shade consists simply of the edges which divide its faces in the light from those in the dark; that is, it consists of the shade lines (21) of the body. These may in many cases be found by simple inspection.

161. *Rays of light* are here assumed to be *parallel straight lines*, as they practically are when proceeding from a very distant source, as the sun, to any terrestrial object.

1*st*. It will be thus observed that the shadow of a vertical edge *ab*, Pl. IX., Fig. 78, of the body of the house will be a vertical line, *a'b'*, on the front wall of the wing behind it; that the shadow

of a horizontal line, as *bc*—the arm for a swinging sign—which is parallel to the wing wall, will be a horizontal line, *b'c'*, parallel to *bc;* that a horizontal line, *bc*, which is perpendicular to the wing wall, will have an oblique shadow, *cb'*, on that wall, commencing at *c*, where the line pierces the wing wall, and ending at *b'*, where a ray of light through *b* pierces the wing wall; and finally that the shadow of a *point*, *b*, is at *b'* where the ray *bb'*, through that point, pierces the surface receiving the shadow.

2*d*. Passing now to Pl. IX., Fig. 79, which represents a chimney upon a flat roof, we observe that the shadows of *bc* and *cd*—lines parallel to the roof—are *b'c'* and *c'd'*, lines equal to, and parallel to, the lines *bc* and *cd;* and that the shadows of *ab* and *ed* are *ab'* and *ed'*—similar to the shadow *cb'* in Fig. 78—*i. e.* commencing at *a* and *e*, where the lines casting them meet the roof, and ending at *b'* and *d'*, where *rays* through *b* and *d* meet the roof.

162. The facts just noted may be stated as *elementary general principles* by means of which many simple problems can be solved.

1*st*. The shadow of a point on any surface, is where a ray of light through that point meets that surface.

2*d*. The shadow of a straight line, upon a plane parallel to it, is a parallel straight line.

3*d*. In like manner, the shadow of a circle upon a plane parallel to it is an equal circle; whose *centre*, only, therefore need be found.

4*th*. The shadow of a line upon a plane to which it is perpendicular, will coincide with the projection of a ray of light upon that plane. Thus *ba*, Fig. 78, being perpendicular to H, its shadow upon that plane coincides with the horizontal projection, *aa'*, of the ray of light *bb'*. Likewise *bc*, being perpendicular to the vertical plane *cb'a'*, its shadow coincides with *cb'*, the projection of *bb'* on that plane.

5*th*. The shadow of a line upon a surface may be said to begin where that line meets that surface, either or both being produced, if necessary. See *a*, Fig. 79.

6*th*. When the shadow, as *aa'b'*, of a line, as *ab*, Fig. 78, falls upon two surfaces which intersect, the partial shadows, as *aa'* and *b'a'*, meet this intersection at the same point, as *a'*.

7*th*. The shadow of a straight line, upon a curved surface, or of a circle, otherwise than as in (3*d*) will generally be a curve, whose points, separately found by (1*st*), must then be joined.

163. In applying these principles to the construction of shadows, three things must evidently be given, viz.—

1*st*. The body casting the shadow.

2*d*. The surface receiving the shadow.

3*d*. The direction of the light.

And these must be given, not in reality, but in *projection*.

164. The light is, for convenience and uniformity, usually assumed to be in such a direction that *its projections* make angles of 45° with the ground line (16). For the *direction of the light itself*, corresponding to these projections of it, see Pl. IX., Fig. 80.

Let a cube be placed so that one of its faces, $L'L_1b$, shall coincide with a vertical plane, and another face, $L''aL_1$, with the horizontal plane. The diagonals, $L'L_1$ and $L''L_1$, of these faces, will be the *projections* of a ray of light, and the diagonal, LL_1, of the cube will be the ray itself; for the point of which L' and L'' are the projections must be in each of the projecting perpendiculars $L'L'$ and $L''L$; hence at L, their intersection. Now the right-angled triangles, $LL''L_1$ and $LL'L_1$, are equal, but not isosceles; hence the angles LL_1L'' and LL_1L', which the ray itself, LL_1, makes with the planes of projection are equal, but *less* than 45°. Again: in the triangle LL_1G, the angle LL_1G is that made by the ray LL_1 with the ground line, and is *more* than 45°.

165. Having now stated the elementary facts and principles concerning *shadows themselves*, we proceed to show how to find their *projections*, by which they are represented.

By (162) we have first to learn how to find where a given line, as a ray of light, pierces a given surface. It will be sufficient here to show how to construct the points in which a line pierces the planes of projection.

Preliminary Problems.

PROBLEM 1*st*. *To find where a line drawn through a given point, pierces the horizontal plane.*

The construction is shown pictorially in Fig. 1, and in actual projection in Fig. 2.

Let A, Fig. 1, be the given point, whose projections are represented by a and a'. If a line passes through a point, its projections will pass through the projections of that point; therefore if AB be the line, ab and $a'b'$ will be its projections.

Now, *first*, the *required* point, C, being in the horizontal plane,

its vertical projection, c', will be on the *ground line; second*, the same point being a point of the given line, its vertical projection will be on the vertical projection, $a'b'c'$, of the line, hence at the intersection, c', of that projection with the ground line; *third*, the *horizontal* projection, c, of the point, which is also the point itself, C, will be where the perpendicular to the ground line from c' meets abc, the horizontal projection of the line.

This explanation may be condensed into the following rule.

1*st.* Note where the vertical projection of the given line, produced if necessary, meets the ground line.

2*d.* Project this point into the horizontal projection of the line; which will give the required point.

Fig. 2, shows the application of this rule in actual projection. $ab—a'b'$ is the given line, and by making the construction, we find c, where it pierces the horizontal plane.

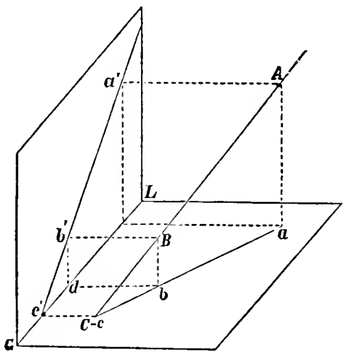


Fig. 1.

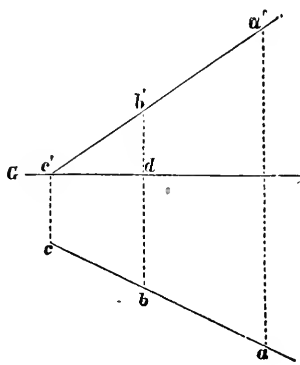


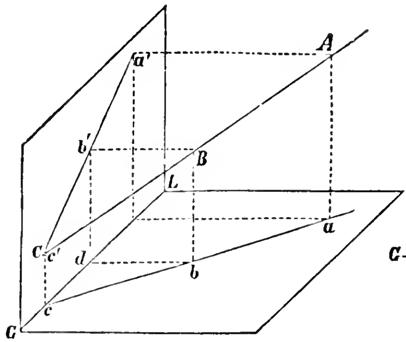
Fig. 2.

Example.—If bd (both figures) had been less than $b'd$, the line would have pierced the horizontal plane *behind* the ground line, that is in the *horizontal* plane produced *backwards*. Let this construction be made, both pictorally, and in projection.

Problem 2*d*.—*To find where a line, drawn through a given point, pierces the vertical plane.*

See Figs. 3 and 4. The explanation is entirely similar to the preceding. The required point, c', being somewhere in the vertical plane, its horizontal projection, c, will be on the ground line. It being also on the given line, its horizontal projection is on the horizontal projection, abc, of the line. Hence, at once, the rule

1st. Note where the *horizontal* projection of the given line meets the ground line.



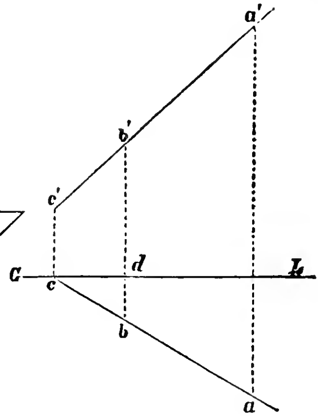


Fig. 3. Fig. 4.

2d. Project this point into the vertical projection of the line, which will give $c' = C$, the vertical projection of the point, which is also the point itself.

Example.—Here likewise, if $b'd$ be *less* than bd, the line will pierce the vertical plane *below* the ground line, that is in the vertical plane extended *downward*. Let the construction be made.

166. The ground line is, really, the *horizontal trace* of the vertical plane; also the *vertical trace* of the horizontal plane. Hence the traces of any other horizontal or vertical planes may be called the *ground lines of those planes;* and therefore the preceding constructions may be applied to finding the shadows on such planes, as will shortly be seen in the solution of problems.

167. In respect to the remaining principles of (162) it only remains to note that, as two points determine a straight line, it is sufficient to find the shadow of *two points* of a straight line, when its shadow falls on a plane. But if the *direction* of the shadow is known as in (162—2*d*, 4*th*) or if we know where the line meets a *plane* surface receiving the shadow, (162—5*th*) it will be sufficient to construct *one point* of the shadow.

§ II.—*Practical Problems.*

168. Prob. 1. *To find the shadow of a vertical beam, upon a vertical wall.* Pl. IX., Fig. 81. Let AA′ be the beam, let the

vertical plane of projection be taken as the vertical wall, and let the light be indicated by the lines, as *ab—a″b′*. The edges which cast the visible shadow are *a—a′a″*; *ac—a″*; and *ce—a″e′*. The shadow of *a—a′a″* is a vertical line from the point *b′*, which point is where the ray from *a—a″* pierces the vertical plane. *ab—a″b′* pierces the vertical plane in a point whose horizontal projection is *b*. *b′* must be in a perpendicular to the ground line from *b* (Art. 15), and also in the vertical projection, *a″b′*, of the ray, hence at *b′*. *b′b* is therefore the shadow of *a—a″a′*. The shadow of *ac—a″* is the line *b′d′*, limited by *d′*, the shadow of the point *c—a″* (162). The shadow of *ce—a″e′* begins at *d′*, and is parallel to *ce—a″e′*, but is partly hidden.

169. *Execution.*—The boundary of a shadow being determined, its surface is, in practice, indicated by shading, either with a tint of indian ink, or by parallel shade lines. The latter method, affording useful pen practice, may be profitably adopted.

170. Prob. 2. *To find the shadow of an oblique timber, which is parallel to the vertical plane, upon a similar timber resting against the back of it.* Pl. IX., Fig. 82. Let AA′ be the timber which casts a shadow on BB′, which slants in an opposite direction. The edge *ac—a′c′* of AA′, casts a shadow, parallel to itself, on the front face of BB′; hence but one point of this shadow need be constructed. Two, however, are found, one being a check upon the other. Any point, *aa′*, taken at pleasure in the edge *ac—a′c′* casts a point of shadow on the front plane of BB′, whose horizontal projection is *b*, and whose vertical projection (see Prob. 1) must be in the ray *a′b′*, and in a perpendicular to the ground line at *b*; hence it is at *b′*. The shadow of *ac—a′c′* being parallel to that line, *b′d′* is the line of shadow. *d′*, the shadow of *c—c′*, was found in a similar manner to that just described.

It makes no difference that *b′* is not on the actual timber, BB′; for the face of that timber is but a *limited physical plane*, forming a portion of the *indefinite immaterial plane*, in which *bb′* is found; hence the point *b′* is as good for finding the direction of the indefinite line of shadow, *b′d′*, as is *d′*, on the timber BB′, for finding the ***real portion*** only of the line of shadow, viz. the part which lies across BB′. Here, *bd* is taken as a ground line (166).

Observe that the shade line A*b*, of the timber AA′ should end at *bb′*, where the timbers intersect.

Example.—Make the timbers larger, AA′ more nearly horizontal than vertical, and then, in both figures, find the shadow on the plan.

171. PROB. 3. *To find the shadow of a fragment of a horizontal timber, upon the horizontal top of an abutment on which the timber rests.* Pl. IX., Fig. 83. Let AA′ be the timber, and BB′ the abutment. The vertical edge, a—$a''a'$, casts a shadow, ab, in the direction of the horizontal projection of a ray (162, *4th*), and limited by the shadow of the point a—a'. The shadow of a—a' is at bb', where the ray ab—$a'b'$ pierces the top of the abutment; b' being evidently the vertical projection of this point, and b being both in a perpendicular, $b'b$, to the ground line and in ab, the horizontal projection of the ray. The shadow of ad—a' is bb'', parallel to ad—a', and limited by the ray $a'b'$—db''. The shadow of de—$a'e'$ is $b''c$, parallel to de—$a'e'$, and limited by the edge of the abutment (162, *2d*). Here, $a''b'$ is used as a ground line (166).

172. PROB. 4. *To find the shadow of an oblique timber, upon a horizontal timber into which it is framed.* Pl. IX., Fig. 84. The upper back edge, ca—$c'a'$, and the lower front edge through ee', of the oblique piece, are those which cast shadows. By considering the point, c, in the shadow of bc, Fig. 78, it appears that the shadow of ac—$a'c'$, Fig. 84, begins at cc', where that edge pierces the upper surface of the timber, BB′, which receives the shadow. Any other point, as aa', casts a shadow, bb', on the plane of the upper surface of BB′, whose vertical projection is evidently b', the intersection of the vertical projection, $a'b'$, of the ray ab—$a'b'$ and the vertical projection, $e'c'$, of the upper surface of BB′, and whose horizontal projection, b, must be in a projecting line, $b'b$, and in the horizontal projection, ab, of the ray. Likewise the line through ee', and parallel to cb, is the shadow of the lower front edge of the oblique timber upon the top of BB′. This shadow is real, only so far as it is actually on the top surface of BB′, and is visible and therefore shaded, only where not hidden by the oblique piece. Where thus hidden, its boundary is dotted, as shown at ea. The point, bb', is in the plane of the top surface of BB′, produced.

Remark.—Thus it appears that when a line is oblique to a plane containing its shadow, the *direction* of the shadow is unknown till found. Let this and the following figures be made much larger.

EXAMPLES.—*1st.* Find the shadow when the oblique timber is more nearly vertical than horizontal.

2d. Let the oblique timber ascend to the right.

173. PROB. 5. *To find the shadow of the side wall of a flight of steps upon the faces of the steps.* Pl. IX., Fig. 85. The steps can be easily constructed in good proportion, without measure

ments, by making the height of each step two-thirds of its width, taking four steps, and making the piers rectangular prisms.

The edges, $aa''—a'$ and $a—ra'$, of the left hand side wall are those which cast shadows on the steps.

The former line casts *horizontal* shadows, as $bb''—b'$, parallel to itself, on the *tops* of the steps (162, 2*d*), and shadows, as $bc''—b'c'$, on the fronts of the steps, in the direction of the vertical projection, $a'd'$, of a ray of light (162, 4*th*)—from the upper step down to the shadow of the point aa'. Likewise, the edge $a—ra'$ casts *vertical* shadows, as $g—g'h'$, on the *fronts* of the steps (161), and shadows on their tops, parallel to ad, the horizontal projection of a ray (162, 4*th*) from the lower step, up to dd', the shadow of aa'; which is therefore, where the shadows of $aa''—a'$, and $a—ra'$, meet. $a'd'$ is the vertical projection of *all* rays through points of $aa''—a'$; hence project down b', etc., to find the parallel shadows, $b''b$, etc. Likewise project up g, etc., to find the shadows, $g'h'$, etc., ad being the horizontal projection of all rays through points of $a—ra'$.

EXAMPLES.—1*st*. Vary the *proportions of the steps* and the *direction of the light*; and in each case, find the shadows, as above.

2*d*. By *preliminary problems* 1*st* and 2*d*, find *directly* where the ray through aa' pierces the steps, only remembering that the projections, d and d', must be on the same surface.

3*d*. Let the piers be cut off by a plane parallel to that of the front edges of the steps. (Use an end elevation.)

174. PROB. 6. *To find the shadow of a short cylinder, or washer, upon the vertical face of a board.* Pl. IX., Fig. 86. Since the circular face of the washer is parallel to the vertical face of the board, BB′, its shadow will be an equal circle (161, 3*d*), of which we have only to find the centre, OO′. This point will be the shadow of the point CC′ of the washer, and is where the ray CO—C′O′ pierces the board BB′. The elements, $rv—r'$ and $tu—t'$, which separate the light and shade of the cylindrical surface, have the tangents $r'r''$ and $t't''$ for their shadows. These tangents, with the semicircle $t''r''$, make the complete outline of the required shadow.

175. PROB. 7. *To find the shadow of a nut, upon a vertical surface, the nut having any position.* Pl. IX., Fig. 86. Let $a'c'e'—ace$ be the projections of the nut, and BB′ the projections of the surface receiving the shadow. The edges, $a'c'—ac$ and $ce—c'e'$, of the nut cast shadows parallel to themselves, since they are parallel to the surface which receives the shadow. $a'n'—an$

are the two projections of the ray which determines the point of shadow, nn'; $c'o'$—co are the projections of the ray used in finding oo', and $e'r'$—er is the ray which gives the point of shadow rr'. The edges at aa' and ee', which are perpendicular to BB′, cast shadows $a'n'$ and $e'r'$, in the direction of the projection of a ray of light on BB′. (See cb', the shadow of cb, Pl. IX., Fig. 78.)

176. Prob. 8. *To find the shadow of a vertical cylinder, on a vertical plane.* Pl. IX., Fig. 87. The lines of the cylinder, CC′, which cast visible shadows, are the element a—$a''a'$, to which the rays of light are tangent, and a part of the upper base. The shadow of a—$a''a'$, is gg', found by the method given in Prob. 1. At g', the curved shadow of the upper base begins. This is found by means of the shadows of several points, bd', cc', dd', &c. Each of these points of shadow is found as g was, and then they are connected by hand, or by the aid of the curved ruler.

It is well to construct one invisible point, as u', of the shadow, to assist in locating more accurately the visible portion of the curved shadow line.

177. Prob. 9. *To find the shadow of a horizontal beam, upon the slanting face of an oblique abutment.* Pl. IX., Fig. 88. The simple facts illustrated by Pl. IX., Figs. 78–79, have no reference to the case of surfaces of shadow, other than vertical or horizontal. But they illustrate the fact that the point where a shadow, as aa', Pl. IX., Fig. 78, on one surface, meets another surface, is a point (a') of the shadow $a'b'$ upon that surface. Thus this problem may be solved in an elementary manner by proceeding indirectly, *i. e.*, by finding the shadows on the horizontal top of the abutment and on its horizontal base. The points where these shadows meet the front edges of this top and this base, will be points of the shadow on the slanting face, que.

By Prob. 3 is found gc, the shadow of the upper back edge, ax—$a'x'$, of the timber, AA′, upon the top of the abutment. c, the point where it meets the front edge, ec, is a point of the shadow of AA′ on the inclined face. By a similar construction with any ray, as bp—$b'p'$, is found qp, the shadow of $c'b'$—db upon the base of the abutment; and q, where it intersects nq, is a point of the shadow of db—$c'b'$ on the face, que. The point, dd', where the edge, db—$c'b'$, meets dc, is another point of the shadow of that edge; hence dq—$d'q'$ is the shadow of the front lower edge, db—$c'b'$, on the inclined face of the abutment. The line through cc', parallel to dq—$d'q'$, is the shadow of the upper back edge, ax—$a'x'$, and completes the solution.

178. Prob. 10. *To find the shadow of a pair of horizontal timbers, which are inclined to the vertical plane, upon that plane.* Pl. IX., Fig. 89. Let the given bodies be situated as shown in the diagram. In the elevation we see the thickness of one timber only, because the two timbers are supposed to be of equal thickness and halved together. As neither of the pieces is either parallel or perpendicular to the vertical plane, we do not know, in advance, the direction of their shadows. It will therefore be necessary to find the shadows of two points of one edge, and one, of the diagonally opposite edge, of each timber. The edges which cast shadows are $ac—a'c'$ and $ht—e'h'$, of one timber, and $ed—e'k$ and $mv—a'm''$ of the other. All this being understood, it will be enough to point out the shadows of the required points, without describing their construction. (See bb', Fig. 81.) bb' is the shadow of aa', and dd' is the shadow of cc'; hence the shadow line $b'—d$ is determined. So uu' is the shadow of hh'; hence the shadow line $u'w'$ may be drawn parallel to $b'd'$. Similarly, for the other timber, ff' is the shadow of ee', and oo' of mm'. One other point is necessary, which the student can construct. The process might be shortened somewhat by finding the shadows of the points of intersection, p and r, which would have been the points p'' and r'' of the intersection of the shadows, and thus, points common to both shadows.

179. Prob. 11. *To find the shadow of a pair of horizontal timbers, which intersect as in the last problem, upon the inclined face of an oblique abutment.* Pl. IX., Fig. 90. This problem is so similar to Prob. 9, that we only note, as a key to it, that sd, corresponding to pq, Fig. 88, is the auxiliary shadow of the edge, $Ab—n'b'$, upon a horizontal plane $s'd'$, cutting the line sb from the face C′ of the abutment, and thence giving a point ss', intersection of sd, parallel to Ab, with sb of the shadow of $Ab—n'b'$ on C′.

The following examples of the very useful *method of auxiliary shadows* (162, 6*th*), and Prob. 9, are here briefly added:

Examples.—1st. *To find the shadow of an abacus of any form, upon a conical column.* Pl. IX., Fig. 88a. Circles OA and OB, with A′B′G′ represent the conical pillar; and circle OC, with C′D′B′, its cylindrical cap, or abacus. Then P′Q′ is a horizontal plane, cutting from the pillar the circle P′Q′, of radius $OB=n$P; and pierced by the ray $Oa—O'a'$ at $a'a$; centre of the circle, of radius $Od=O'D'$. Then d, projected at d', and intersection of the circle Od with the circle OQ, is a point of the shadow of circle OC—O′D′ of the abacus, upon the pillar; since the circle Od is the auxiliary

shadow of the circle OC—O′D′ upon the plane P′Q′ (162, 3*d*). Other points may be likewise found.

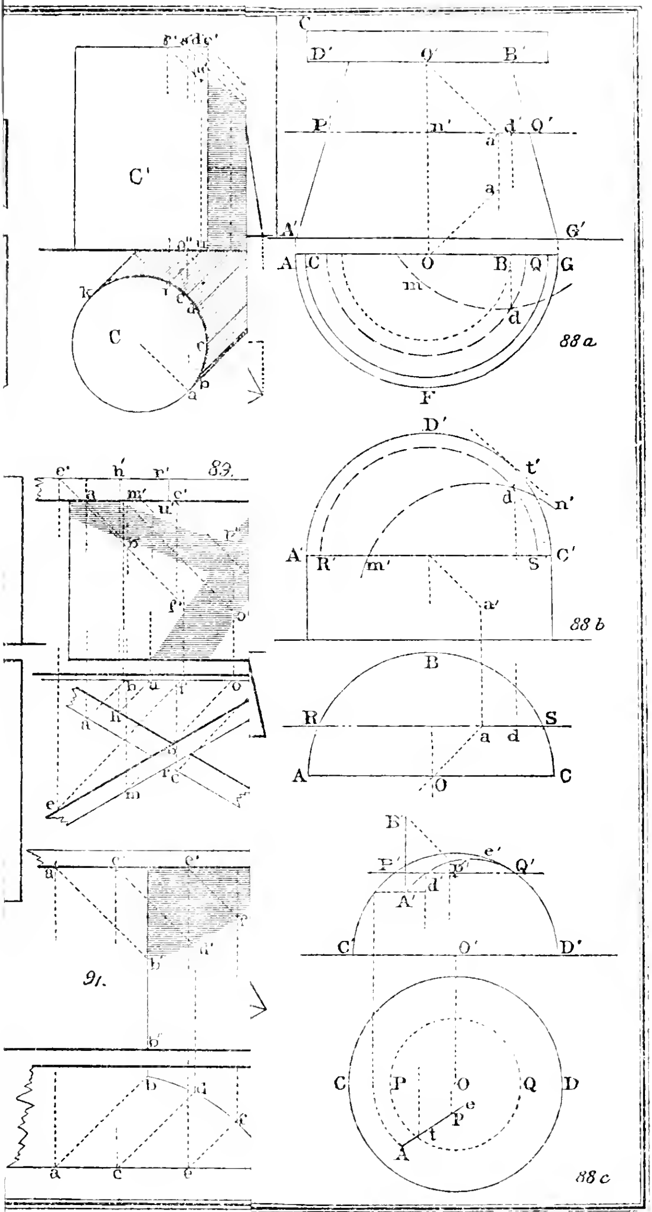
2d. *To find the shadow of the front circle of a niche, upon its own spherical surface.* Pl. IX., Fig. 88b. ABC—A′D′C′ is the quarter sphere; which surmonts the vertical cylindrical part of the niche below the line ABC— A′O′C. Then the ray O*a*—O′*a*′ meets any plane RS, parallel to the face, AOC—A′D′C′, of the niche, at *aa*′; giving circle *a*′,*m*′*n*′= circle O′A′, for the auxiliary shadow of circle O′A′ upon the plane RS (162, 3*d*), circle *a*′, *m*′*n*′ then cuts circle RS—R′*d*′S′, cut from the spherical part of the niche by the plane RS, at *d*′*d*′, a required point of shadow. Find other points likewise, and join them. The tangent ray at *t*′ gives that point.

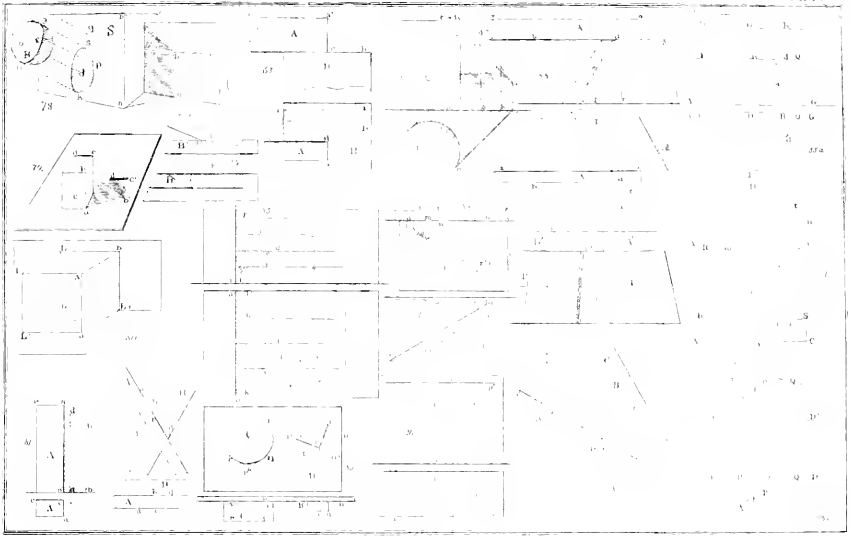
3d. *To find the shadow of a vertical staff. upon a hemi spherical dome.* Pl. IX., Fig. 88c. Circle OC, with C′P′D′ is the hemisphere, and A—A′B′ the staff. By (162, 4*th*) A*e* the horizontal projection of a ray, is also the shadow of A—A′B′ upon the assumed horizontal plane P′Q′, and cutting from the hemisphere the circle P′Q′—PQ. Then *dd*′, intersection of A*e* with circle PQ—P′Q′, is one point of the required shadow of A—A′B′ upon the dome.

180. Prob. 12. *To find the shadow of the floor of a bridge upon a vertical cylindrical abutment.* Pl. IX., Fig. 91. The line *ag*—*a*′*g*′ is the edge of the floor which casts the shadow. *bdg*—*b*′*e*′*g*′*n* is the concave vertical abutment receiving the shadow. *gg*′, where the edge *ag*—*g*′*a*′ of the floor meets this curved wall, is one point of the shadow. *f* is the horizontal projection of the point where the ray, *ef*—*e*′*f*′, meets the abutment; its vertical projection is in *e*′*f*′, the vertical projection of the ray, and in a perpendicular to the ground line, through *f*, hence at *f*′. Similarly we find the points of shadow, *dd*′ and *bb*′, and joining them with *f*′ and *g*′, have the boundary of the required shadow. Observe, that to find the shadow on any particular vertical line, as *b*—*b*″*b*′, we draw the ray in the direction *b*—*a*; then project *a* at *a*′, &c.

Remark.—The student may profitably exercise himself in changing the positions of the given parts, while retaining the methods of solution now given.

For example, let the parts of the last problem be placed side by side, as two elevations, giving the shadow of a vertical wall on a horizontal concave cylindrical surface; or, let the timbers, Fig. 89, be in vertical planes, and let their shadows then be found on a horizontal surface.


88 a
88 b
88 c
89.
91.


78
79

CHAPTER II.

SHADING.

131. The distinction between a *shade* and a *shadow* is this. A *shadow*, as indicated by the preceding problems, is the portion of a body from which light is excluded by *some other body*. A *shade*, is that portion of the surface of a body from which light is excluded by *the body itself* (158, 159).

The accurate representation of *shades* assists in judging of the *forms* of bodies; that of *shadows* is similarly useful, besides aiding in showing their *relations to surrounding bodies*.

In either case, a *flat tint* mainly shows only *where* the shade, or shadow, is; while *finished shading* helps to show the *form and position* of the body.

182. EXAMPLE 1°. *To shade the elevation of a vertical right hexagonal prism, and its shadow on the horizontal plane.* Pl. X., Figs. 92 and A. Let the prism be placed as represented, at some distance from the vertical plane, and with none of its vertical faces parallel to the vertical plane. The face, A, of the prism is in the light; in fact, the light strikes it nearly perpendicularly, as may be seen by reference to the plan; hence it should receive a very light tint of indian ink. The left hand portion of the face, A, is made slightly darker than the right hand part, it being more distant; for the reflected rays, which reach us from the left hand portion, have to traverse a greater extent of air than those from the neighborhood of the line tt', and hence are more absorbed or retarded; since the atmosphere is not perfectly transparent. That is, these rays make a weaker impression on the eye, causing the left hand portion of A, from which they come, to appear darker than at tt'

Remarks.—a. It should be remembered that the whole of face A is very light, and the difference in tint between its opposite sides very slight.

b. As corollaries from the preceding, it appears: *first*, that a surface parallel to the vertical plane would receive a uniform tint throughout; and, *second*, that of a series of such surfaces, all of

which are in the light, the one nearest the eye would be lightest, and the one furthest from the eye, darkest.

c. It is only for great differences in distance that the above effects are manifest in nature; but drawing by projections being artificial, both in respect to the shapes which it gives in the drawings, and in the absence of surrounding objects which it allows, we are obliged to exaggerate natural appearances in some respects, in order to convey a clearer idea of the forms of bodies.

d. The mere manual process of shading small surfaces is here briefly described. With a sharp-pointed camel's-hair brush, wet with a very light tint of indian ink, make a narrow strip against the left hand line of A, and soften off its edge with another brush slightly wet with clear water. When all this is dry, commence at the same line, and make a similar but wider strip, and so proceed till the whole of face A is completed, when any little irregularities in the gradation of shade can be evened up with a delicate sable brush, *damp* only with very light ink.

183. Passing now to face B, it is, as a whole, a little darker than A, because, as may be seen by reference to the plan, while a beam of rays of the thickness *np* strikes face A, a beam having only a thickness *pr*, strikes face B; i. e., we assume, *first*, that the *actual brightness* of a flat surface is proportioned to the number of rays of light which it receives; and, *second*, that its *apparent brightness* is, other things being the same, proportioned to its actual brightness. Also the part at $a—a'$ being a little more remote than the line $t—t'$, the part at $a—a'$ is made a very little darker.

184. The face C is very dark, as it receives no light, except a small amount by reflection from surrounding objects. This side, C, is *darkest* at the edge $a—a'$ which is nearest to the eye. This agrees with experience; for while the shady side of a house near to us appears in strong contrast with the illuminated side, the shady side of a remote building appears scarcely darker than the illuminated side. *This fact is explained as follows:* The air, and particles floating in it, between the eye and the dark surface, C, are in the light, and reflect some light in a direction from the dark surface C to the eye; and as the air is invisible, these reflections *appear* to come from that dark surface. Now the more distant that surface is, the greater will be the body of illuminated air between it and the eye, and therefore the greater will be the amount of light, apparently proceeding from the surface, and its consequent apparent brightness. That is, the more distant a surface in the dark is, the

lighter it will appear. It may be objected that this would make out the *remoter* parts of illuminated surfaces as the lighter parts. But not so; for the air is a nearly perfect transparent medium, and hence reflects but little, compared with what it transmits to the opaque body; but being not quite transparent, it absorbs the reflected rays from the distant body, in proportion to the distance of that body, making therefore the remoter portions darker; while the *very weak* reflections from the shady side are *reinforced*, or replaced, by more of the *comparatively stronger* atmospheric reflections, in case of the remote, than in case of the near part of that shady side. Thus is made out a consistent theory.

In relation, now, to the shadow, it will be lightest where furthest from the prism, since the atmospheric reflections evidently have to traverse a less depth of darkened air in the vicinity of *de*—*d'* than near the *lower* base of the prism at *abc*.

185. Ex. 2°. *To shade the elevation of a vertical cylinder.* Pl. X., Fig. 93. Let the cylinder stand on the horizontal plane. The figures on the elevation suggest the comparative depth of color between the lines adjacent to the figures. The reasons for so distributing the tints will now be given. See also Fig. B.

The darkest part of the figure may properly be assumed to be that to which the rays of light are tangent; viz., the vertical line at *tt'*, from which the tint becomes lighter in both directions.

The lightest line is that which reflects most light to the eye. Now it is a principle of optics that the incident and the reflected ray make equal angles with a perpendicular to a surface. But *n*C is the incident ray to the centre, and C*e* the reflected ray from C to the eye (12). Hence *d*, which bisects *ne*, shows where the incident ray, *ed*, and reflected ray, *dq*, make equal angles with the perpendicular (normal) *d*C, to the surface. Hence *d*—*d'* is the lightest element.

186. *Remark.*—The question may here arise, "If all the light that is reflected towards the eye is reflected from *d*—as it appears to be—how can any other point of the body be seen?" To answer this question requires a notice—*first*, of the difference between polished and dull surfaces; and, *second*, between the case of light coming *wholly* in one direction, or *principally* in one direction. If the cylinder CC', considered as perfectly polished, were deprived of all reflected light from the air and surrounding objects, the line at *d*—*d'* would reflect to the eye all the light that the body would remit towards the eye, and would appear as a line of brilliant light, while other parts, remitting no light whatever, would be totally

invisible. Let us now suppose a reflecting medium, though an imperfect one, as the atmosphere, to be thrown around the body. By reflection, every part of the body would receive some light from all directions, and so would remit some light to the eye, making the body visible, though faintly so. But no body has a polish that is absolutely perfect; rather, the great majority of those met with in engineering art have entirely dull surfaces. Now a dull surface, greatly magnified, may be supposed to have a structure like that shown in Pl. X., Fig. 97, in which many of the asperities may be supposed to have one little facet each, so situated as to remit to the eye a ray received by the body directly from the principal source of light.

187. Having thus shown how any object placed before our eyes can be seen, we may proceed with an explanation of the distribution of tints. *b* is midway between *d* and *t*. At *b*, the ink may be diluted, and at *e—e'*, much more diluted, as the gradation from a faint tint at *e* to absolute whiteness at *d* should be without any abrupt transition anywhere.

The beam of incident rays which falls on the segment *dn*, is broader than that which strikes the equal segment *de;* hence the segment *nd* is, on the elevation, marked 5, as being the lightest band which is tinted at all. The segment *nr*, being a little more obliquely illuminated, is less bright, and in elevation is marked 3, and may be made darkest at the left hand limit. Finally, the segment *rv* receives about as much light as *rn*, but reflects it within the very narrow limits, *s*, hence appears brighter. This condensed beam, *s*, of reflected rays would make *rv* the lightest band on the cylinder, but for two reasons; *first*, on account of the exaggerated effect allowed to increase of distance from the eye; and, *second*, because some of the asperities, Fig. 97, would obscure some of the reflected rays from asperities still more remote; hence *rv* is, in elevation, marked 4, and should be darkest at its right hand limit.

188. The process of shading is the same as in the last exercise. Each stripe of the preliminary process may extend past the preceding one, a distance equal to that indicated by the short dashes at the top of Fig. 94. When the whole is finished, there should be a uniform gradation of shade from the darkest to the lightest line, free from all sudden transitions and minor irregularities.

189. Ex. 3°. *To shade a right cone standing upon the horizontal plane, together with its shadow.* Pl. X., Fig. 95. The shadow of the cone on the horizontal plane, will evidently be bounded

by the shadows of those lines of the convex surface, at which the light is tangent. The vertex is common to both these lines, and casts a shadow, v'''. The shadows being cast by straight lines of the conic surface, are straight, and their extremities must be in the base, being cast by lines of the cone, which meet the horizontal plane in the cone's base; hence the tangents $v'''t$ and $v'''t''$ are the boundaries of the cone's shadow on the horizontal plane, and the lines joining t' and t'' with the vertex are the lines to which the rays of light are tangent; i. e., they are the darkest lines of the shading; hence tv, the visible one in elevation, is to be vertically projected at $t'v'$.

The lightest line passes from vv' to the middle point, y, between n and p in the base. At q and at p a change in the darkness of the tint is made, as indicated by the figures seen in the elevation. In the case of the cone, it will be observed that the various bands of color are triangular rather than rectangular, as in the cylinder; so that great care must be taken to avoid filling up the whole of the upper part of the elevation with a dark shade. See Pl. X., Fig. C.

190. Ex. 4°. *To shade the elevation of a sphere.* Pl. X., Fig. 96. It is evident that, if there be a system of parallel rays, tangent to a sphere, their points of contact will form a great circle, perpendicular to these tangents; and which will divide the light from the shade of the sphere. That is, it will be its circle of shade. Each point of this circle is thus the point of contact of one tangent ray of light. If now, parallel planes of rays, that is, planes parallel to the light, be passed through the sphere, each of them will cut a circle, great or small, from the sphere, and there will be two rays tangent to it on opposite sides, whose points of contact will be points of the circle of shade.

In the construction, these parallel planes of rays will be taken perpendicular to the vertical plane of projection.

Next, let us recollect that always, when a line is parallel to a plane, its projection on that plane will be seen in its true direction. Now BD′ being the direction of the light, as seen in elevation, let BD′ be the trace, on the vertical plane of projection—taken through the centre of the sphere—of a new plane perpendicular to the vertical plane, and therefore parallel to the rays of light. The projection of a ray of light on this plane, BD′, will be parallel to the ray itself, and therefore the angle made by this projection with the trace BD will be equal to the angle made by the ray with the vertical plane. But, referring to Pl. IX., Fig. 80, we see that in the triangle

$LL'L_1$, containing the angle LL_1L' made by the ray LL_1 with the vertical plane, the side $L'L_1$ is the hypothenuse of the triangle $L'b\,L_1$, each of whose other sides is equal to LL'. Hence in Pl. IX., Fig. 96, take any distance, B*c*, make AB perpendicular to BD', and on it lay off BD = B*c*, then make BD' = D*c*, join D and D', and DD' will be the projection of a ray upon the plane BD', and BD'D will be the true size of the angle made by the light with the vertical plane; it being understood that the plane BD', though in space perpendicular to the vertical plane, is, in the figure, represented as revolved over towards the right till it coincides with the vertical plane of projection, and with the paper.

191. We are now ready to find points in the curve of shade. *oo'* is the vertical projection of a small circle parallel to the plane BD', and also of its tangent rays. The circle *og'o'*, on *oo'* as a diameter, represents the same circle revolved about *oo'* as an axis and into the vertical plane of projection. Drawing a tangent to *og'o'*, parallel to DD', we find *g'*, a visible point of the curve of shade, which, when the circle revolves back to the position *oo'*, appears at *g*, since, as the axis *oo'* is *in* the vertical plane, an arc, *g'g*, described about that axis, must be *vertically* projected as a straight line. (See Art. 31.)

In a precisely similar manner are found *h*, *k*, *m*, and *f*. At A and B, rays are also evidently tangent to the sphere. Through A, *f*, &c., to B the visible portion of the curve of shade may now be sketched.

192. The most highly illuminated point is 90° distant from the great circle of shade; hence, on ABQ, the revolved position of a great circle which is perpendicular to the circle of shade, lay off *k'*Q = *q*B = the chord of 90°, and revolve this perpendicular circle back to the position *qq'*, when Q will be found at *r'*. But the brilliant point, as it appears to the eye, is not the one which *receives* most light, but the one that *reflects* most, and this point is midway, in space, between *r'* and *r*, i. e. at P, found by bisecting QB, and drawing RP; for at R the incident ray whose revolved position is parallel to Q*r* or DD', and the reflected ray whose revolved position is parallel to *r*B, make equal angles with R'R, the perpendicular (normal) to the surface of the sphere. See Pl. X., Fig. D.

193. In regard now to the second general division of the problem —the distribution of tints; a small oval space around P should be left blank. The first stripe of dark tint reaches from A to B, along the curve of shade, and the successive stripes of the same tint extend to B*q*A on one side of B*k*A, and to *oew* on the other. Then take

a lighter tint on the lower half of the next zone, and a still lighter one on its upper half (2) and (3). In shading the next zone, use an intermediate tint (3–4), and in the zone next to P a very light tint on the lower side (4), and the lightest of all on the upper side (5). After laying on these preliminary tints, even up all sudden transitions and minor irregularities as in other cases.

194. Ex. 5°. **Shades and Shadows on a Model.** Pl. XI., Figs. 1 and 2. *General Description.*—This plate contains two elevations of an architectural Model. It is introduced as affording excellent practice in tinting and shading large surfaces, and useful elementary studies of shadows. The construction of these elevations from given measurements is so simple, that only the base and several centre lines need be pointed out.

QR is the ground line. ST is a centre line for the flat topped tower in Fig. 1. UV is a centre line for the whole of Fig. 2, except the left-hand tower and its pedestal. WX is a centre line for the tower through which it passes. YZ is the centre line for the roofed tower in Fig. 1. The measurements are recorded in full, referred to the centre lines, base line, and bases of the towers, which are the parts to be first drawn.

Graphical construction of the shadows.

1°. The roof, D—D′D″, casts a shadow on its tower. The point, EE′, casts a shadow where the ray, E*e*, pierces the side of the tower. *e* is one projection of this point; *e*′, the other projection of the same point, is at the intersection of the line *ee*″*e*′ with the other projection, E′*e*′, of the ray. The shadow of a line on a parallel plane (162) is parallel to itself, hence *e*′*f*′, parallel to E′F′, is the shadow of E′F′.

The shadow of DE—D′E′ joins *e*′ with the shadow of D—D′. The point *d*, determined by the ray D*d*, is one projection of the latter shadow; the other projection, *d*′, is at the intersection of *dd*″*d*′ with the other projection, D′*d*′, of the ray. *d*′ is on the side of the tower, produced, hence *e*′*d*′ is only a real shadow line from *e*′ till it intersects the edge of the tower.

Remembering that the direction of the light is supposed to change with each position of the observer, so that as he faces each side of the model, in succession, the light comes from left to right and from behind his left shoulder, it appears that the point, DD″, casts a shadow on the face of the tower, seen in Fig. 2, and that D″*d*‴ will be the position of the ray, through this point, on Fig.

1. The point d''' is therefore one projection of the shadow of DD″. The other is at d'''', the intersection of the lines $d''d'''$ with Dd'''', the other projection of the ray. Likewise EE″ casts a shadow, $e'''e''''$, on the same face of the tower, produced. DD‴, being parallel to the face of the tower now being considered, its shadow, $d''''q$, is parallel to it. The line from d'''' towards e'''', till the edge of the tower, is the real portion of the shadow of DE—D″E″.

From the foregoing it will be seen how most of these shadows are found, so that each step in the process of finding similar shadows will not be repeated.

2°. The body of the building—or model—casts a shadow on the roofed tower, beginning at AA′ (161). The shadow of BB′ on the side of this tower is bb', found as in previous cases, and A′b' is the shadow of AB—A′B′. From b' downwards, a vertical line is the shadow of the vertical corner edge of the body of the model upon the parallel face of the tower.

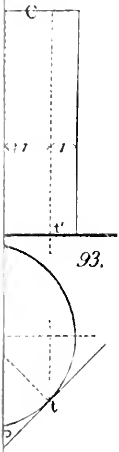
3°. The line CC″—C′, which is perpendicular to the side of the roofed tower, casts a shadow, C′c', in the direction of the projection of a ray of light on the side of the tower.

4°. In Fig. 1, a similar shadow, $s't'$, is cast by the edge s'—ss' of the smaller pedestal.

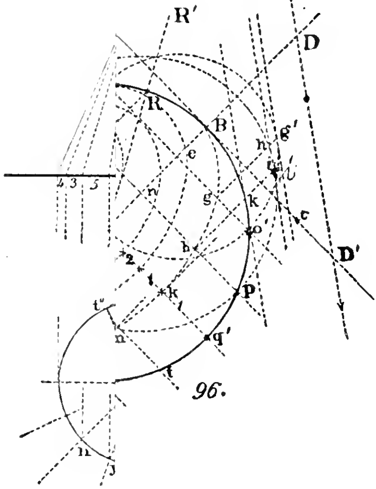
5°. In Fig. 2, is visible the curved shadow, $c''rg$, cast by the vertical edge, at c'', of the tower, on the curved part of the pedestal of the tower. The point g is found by drawing a ray, G′C′—Gg, which meets the upper edge of the pedestal at gC′. The point c'', the intersection of the edge of the tower with the curved part of the pedestal, is another point. Any intermediate point, as r, is found by drawing the ray R′r'; r' is then one projection of the shadow of R′R, and the other is at the intersection of the line $r'r$ with the other projection Rr of the ray. These are all the shadows which are very near to the objects casting them.

6°. The flat topped tower casts a shadow on the roof of the body. The upper back corner, HH′, casts a shadow on the roof, of which h is one projection and h' the other. The back upper edge H—H′I′ being parallel to the roof, the short shadow $h'h''$, leaving the roof at h'', is parallel to H′I′. The left hand back edge HJ—H′J′ casts a shadow on the roof, of which hh' is one point. The point JJ′ casts a shadow jj' on the roof produced. $h'j'$ is therefore a real shadow only till it leaves the actual roof at u.

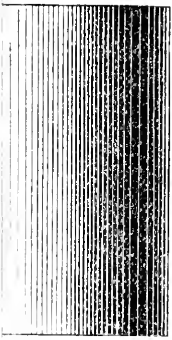
7°. The same tower casts a shadow on the vertical side of the body, of which $j''j'''$, found as in previous cases, and u, are points



93.

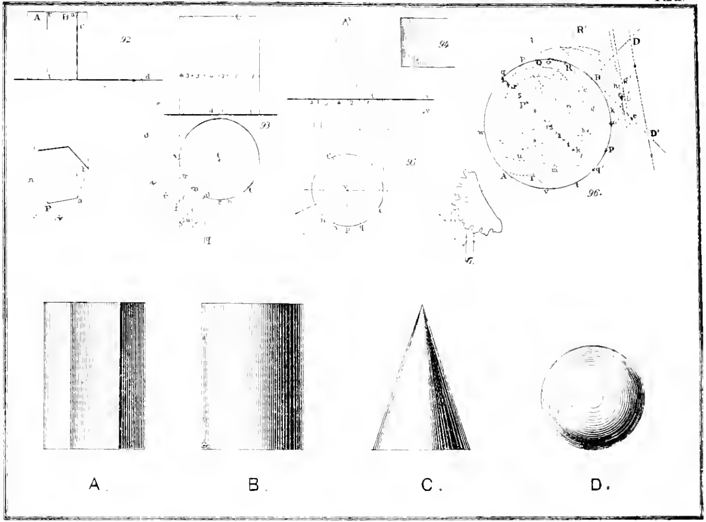


96.



B.

D.



A. B. C. D.

the upper back point, KK′, of the shaft of the tower, casts a shadow, kk', which is joined with j''', giving the shadow of JK—J′K′. From k', $k'l$ is the vertical shadow line of the left-hand back edge of the flat-topped tower on the parallel plane of the side of the body of the model.

8°. The same tower casts a shadow on the curved—cylindrical part of the pedestal. To find the point m', of shadow, draw a ray, MC, Fig. 2, intersecting the upper edge of the pedestal at C, which is therefore one projection of the shadow of the point MM′. The other projection, m', of the same shadow, is at the intersection of the other projection, $M'm'$, of the ray, with the other projection, $m'C'$, of the edge of the pedestal. The point of shadow, nn', cast by the point NN′, is similarly found, and so is the point oo', cast by the point OO′ of the front right-hand edge of the tower. Make $m'm'' = n'o'$, and find intermediate points, $v'v''$, as rr' was found, and the curved shadow on the cylindrical part of the pedestal will then be found.

9°. From n' and o', vertical lines are the shadows of opposite diagonal edges of the tower, on the vertical face of the main pedestal.

10°. This flat-topped tower also casts a shadow on the side of the roofed tower. The right back corner, H—I′, of the top, casts the shadow $h'''h''''$ on the side of the roofed tower, through which the shadow line, $h''''x$ is drawn, parallel to the line H—H′I′ which casts it. The right-hand top line, I′—H, being perpendicular to the plane of the sides of this tower, casts the shadow $h''''i'$ upon it, parallel to the projection of a ray of light. (162.) This shadow line is real, only till it leaves the tower at z;—i' being in the plane of the side of the tower produced—and it completes all the shadows visible in the two elevations.

Errors in Shading, Relative darkness of the light and shade, etc.

195. The most frequent faults to guard against are—1*st*. *A brush too wet, or too long applied to one part of the figure;* giving a ragged or spotty appearance.

2*d*. *Outlines inked in black*, whereas the form and outlines of actual objects are indicated, *not* by black edges, but by contrast of shade only, with edges *lighter* than other parts.

3*d*. *Much too little contrast* between the shading of the parts in

the *light* and those in the dark. The former should generally, as on a cylinder, be *very much* lighter, or not one quarter as dark as the parts in the dark. This is confirmed by examining photographs of objects illuminated in the manner supposed in this chapter.

196. There are *five methods of shading*, as follows :

1*st*. *Softened wet shading*. This is done with a large brush, quite wet, and is applicable to large figures.

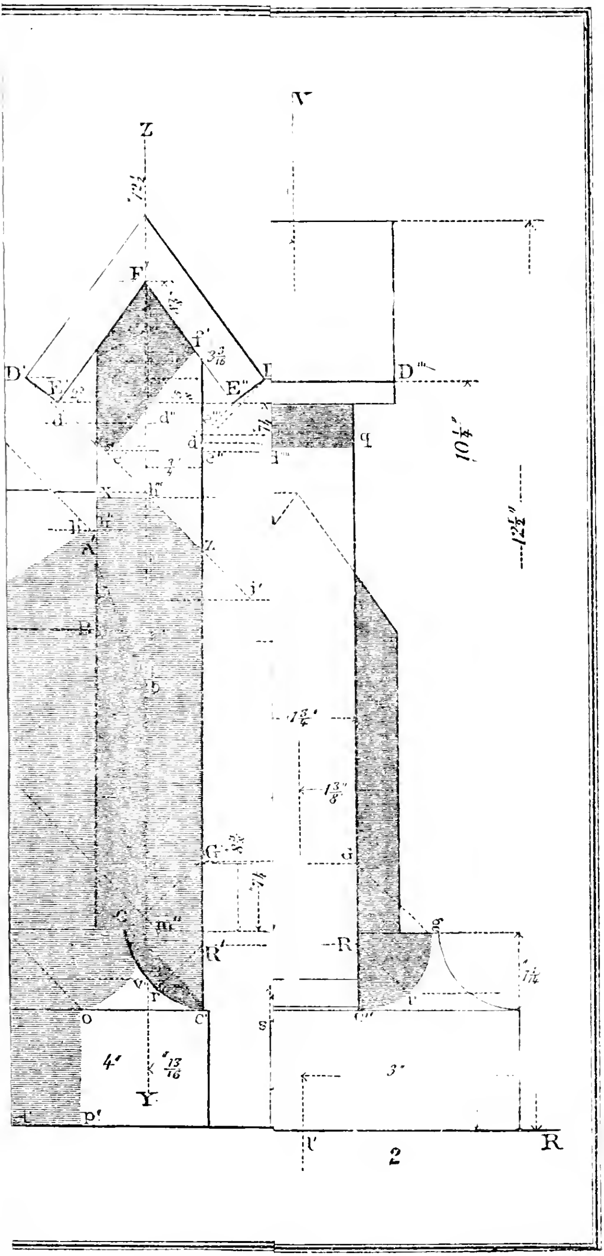
2*d*. *Softened dry shading*. This is done with a brush, nearly dry, and is applicable to figures of the size of those on Pl. X., or not much larger.

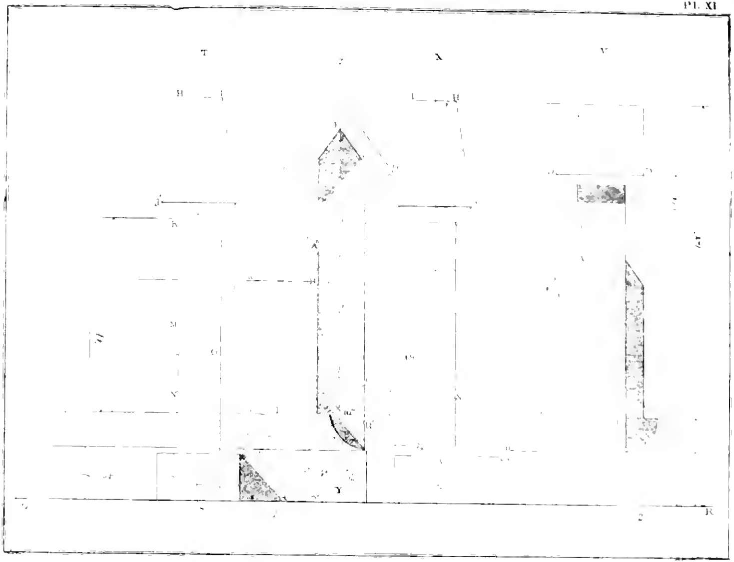
3*d*. *Shading by superposed flat tints*. This method is very neat and effective for large figures, not to be closely examined. It consists of the preliminary stripes of the 2*d* method evenly and smoothly done, *without* softening their edges (183 *d*).

4*th*. *Stippling, or dot shading*. This is done with a fine pen as in making sand in topographical drawing.

5*th*. *Line shading*. This is done by means of lines of graded size and distance apart ; as in wood and lithographic mechanical engravings.

The details of these methods are further explained in my "Drafting Instruments and Operations."





DIVISION FOURTH.

ISOMETRICAL AND OBLIQUE PROJECTIONS.

CHAPTER I.

FIRST PRINCIPLES OF ISOMETRICAL DRAWING.

197. *It is the object of this Division* to explain some methods of making drawings, especially of details and various small work, which combine the intelligibleness of pictorial figures, with the exactness of common projections (Div. I.). Such drawings possess, among others, the advantage of being more readily understood by workmen unacquainted with ordinary projections, than plans and elevations might be.

We shall therefore now explain the methods called Isometrical and Oblique Projections, taking up the former first.

198. *Isometrical projection* is that in which a solid right angle, like that at the corner of a cube, is placed so that the three plane right angles which bound it appear equal in the projection.

The elementary principles of this projection are most simply explained, as follows, by reference to a cube, since this is the simplest possible rectangular body.

For clearness, we have to distinguish in a cube, its *edges*, its *face diagonals*, and its *body diagonals*.

199. Accordingly, let Pl. XII., Fig. 98, represent a cube whose front face is parallel to V. This face will therefore be the only one visible in elevation, and will appear in its real size.

Next suppose the cube to be turned horizontally 45°, when two of its vertical faces will be shown equally, as in Fig. 99, though not in their real size.

Finally, suppose the cube to be turned up at its back corner *dd'*, about any axis as *ef*, parallel to the ground line, until the

top and the two faces shown in Fig. 99 all appear equal, as in Fig. 100. This evidently can be done, for in Fig. 99 the top face is not seen at all in vertical projection: but invert the figure, taking plan for elevation, and it is fully seen, and the vertical faces are unseen: hence there must be one intermediate position, where, as in Fig. 100, the three faces seen in Fig. 99 will be seen equally.

200. *Results.*—1°. The equality of the projections of the three visible *faces* of the cube, when inclined as in Fig. 100, includes the equality of the projections of its *edges*. Hence (Div. I., Art. 11), these edges also being equal in space, are equally inclined to the plane of projection.

2°. The equal projections of the *faces* include the equal projections of their like *angles*. Hence the three angles at C, and the angles equal to them at D, o, p, are angles of 120° each. The remaining angles of the projections of the faces are of 60° each.

3°. Lines like Ce'', $C''D$, $C''f''$, Fig. 100, equal, and equally inclined to a plane, and proceeding from one point, C, must terminate in a plane parallel to the given plane. Hence the three face diagonals $e''f''$, $f''n$, ne', are all parallel to the plane of projection, and therefore appear in their real size. Hence in (199) we might have said, incline the cube, until three visible face diagonals, $b'd'$—BD; $a''b'$—A'B; $a''d'$—A'D, Fig. A, become parallel to the plane of projection, XY. All other lines of Fig. 100 appear less than their real size.

4°. By 3°, $e''f''n$ is evidently an equilateral triangle; and, by 1° and 2°, all the other lines make equal angles of 30° with these. Hence the perimeter $Df''onpe''$ is a regular hexagon, and the projections of the visible foremost, and invisible hindmost corners of the cube coincide at C. Hence the diagonal of the cube, which joins these points, is perpendicular to the plane of projection.

201. *Definitions.* Fig. 100: C is the *isometric centre*, Ce'', Cf'', $C'n$, are the *isometric axes*. Lines parallel to these axes are *isometric lines*; others are *non-isometric lines*. Planes parallel to the faces of the cube are *isometric planes*.

The *edges* of a cube, or similar rectangular body, are the lines

whose dimensions would naturally be desired. Hence it is usual to make Ce'', Cf'', De'', etc., Fig. 100, equal to the edges of the cube itself; as in Fig. 101. Such a figure is, for distinction, called the *Isometrical Drawing* of the cube, and it is the isometrical *projection* of an imaginary larger cube, which is to Fig. 101 as Fig. 99 is to Fig. 100.

202. *Another demonstration.** The first principles of isometrical projection may be developed from the following

PROPOSITION. *The plane which is perpendicular to the body diagonal of a cube, is equally inclined to its faces and edges.*

Let $abcd—a'b'c'd'a''b''c''d''$, Pl. XII., Fig. A, be the plan and elevation of a cube, shown as in Pl. XII., Fig. 99, only that the ground line, GL, is inclined, to permit the construction of the isometrical figure in an upright position, as in Fig. 100, by direct projection from the given elevation.

The body diagonal, $ac—a'c''$, parallel to V, is the common hypothenuse of three equal right-angled triangles, similarly situated relative to the cube. One base of each of these triangles is an edge, beginning at cc''; the other is a face diagonal, beginning at aa'.

Thus, one of these three triangles is $ac — a'c'c''$, which, being parallel to V, shows its real size on V. Another has for its bases the edge $bc—b''c''$ and the face diagonal $ab—a'b''$; and the third has for its bases the edge $dc—d''c''$, and the face diagonal $ad—a'd''$.

Now since these triangles are thus equal, and similarly placed on the cube, their common hypothenuse, the body diagonal $a'c''$, making equal angles with the three face diagonals which meet at aa', also makes equal angles with the faces containing these diagonals.

Hence a plane of projection XY, or V_1, perpendicular to this body diagonal, is equally inclined to the three faces of the cube which meet at aa', or at cc''; which agrees with the enunciation.

From this conclusion follow all the other particulars in the preceding articles.

In making the figure, H and the plane XY (V_1) are both perpendicular to V. Hence $Cn=cc''$; $Do=dd''$; etc.

* This may be omitted at discretion, but may be preferred by Teachers and others, as fresher, and more concise and strictly geometrical.

CHAPTER II.

PROBLEMS INVOLVING ONLY ISOMETRIC LINES.

203. PROB. 1. *To construct the isometrical projections, and drawings of cubes, and a rectangular block cut from a corner of one of them.*

The principles of the last chapter yield several simple constructions, as follows:

204. *First Method.* Draw an equilateral triangle, as $e''f''n$, Fig. 100, each of whose sides shall be equal to a face diagonal of the cube. Then (200, 4°) draw two lines, as e''D and e''C, with the 30° angle of the 30° and 60° triangle, making angles of 30° with each side of the triangle $e''f''n$, at each extremity. These lines will intersect as at C, D, o, etc., forming the isometric *projection* of the cube. By extending Ce'', Cf'', Cn, till equal to the edge of the cube, and joining their new outer extremities, the projection will be converted into the isometric *drawing.*

205. *Second Method.* Fig. 100. Draw the isometric axes indefinitely, Ce'' and Cf'' each making angles of 30° with a horizontal EF, on which lay off half of a face diagonal ef, Fig. 99, each way from C, giving E and F. Then perpendiculars at E and F will limit the right and left axes at e'' and f''. Cn is then made equal to Ce'' or Cf'', and the remaining edges are drawn parallel to these three. From this *projection,* make the *drawing* as in the first method.

206. *Third Method.* Drawing the axes as before, lay off on each the true length of an edge of the cube, and complete the figure as just described. This at once makes the *drawing.*

207. *Fourth Method.* Fig. 101. With centre C and radius equal to an edge of the cube, draw a circle, and in it inscribe a regular hexagon in the position shown, adding the alternate radii Ca, Cb, Cc, which again gives the *drawing* of the cube.

Let a prismatic block be cut from the front corner of this cube. Suppose the edge of the cube to be five inches long, drawn

to a scale of *one-fifth*. Let Ca'=2 inches; Cb'=3 inches; Cc'= 1 inch. Lay off these distances upon the axes, and at a', b', c', draw isometric lines which will be the remaining visible edges of the block.

EXAMPLES.—1*st*. Draw a square panel in each visible face.

2*d*. Draw a square tablet in each visible face.

3*d*. Let a cube $1\frac{1}{2}$ inches on each edge be cut from every visible corner of the cube.

208. PROB. 2. *To find the shade lines on a cube, and the shadows of isometric lines upon isometric planes.*

Three faces of the cube, Pl. XII., Fig. 102, will be illuminated by light passing in the direction LL′, from *a* to *p*. Two of these faces, *aon*C, and *a*C*q*D are visible. The under and right-hand faces, *onp*C, *npq*C, and *pq*DC, are in the dark, hence by the rule (18) the edges *on*, *n*C, C*q* and *q*D are to be inked heavy.

The shadows of *ab*=*oa* produced; and of *ad*=D*a* produced. *ab* is perpendicular to *a*C*q*D, hence (162, 4*th*) its shadow will be in the direction of the projection of the light upon that face. But *aq* is evidently the projection of the ray, LL′ upon *a*C*q*D: and as *a* is where *ab* meets *a*C*q*D (162, 5*th*), *ab′* is the shadow of *ab* on *a*C*q*D.

In like manner, *ad′* is the shadow of *ad* upon *aon*C.

Otherwise. A plane containing *oa* and *ap*, will contain rays through points of *oab*, and will also contain *pq* and will therefore cut *a*C*q*D in the line *aq*. Hence rays from all points of *ab* will pierce *a*C*q*D in *aq*. Hence *ab′* is the shadow of *ab*. Again; D*ap* is the plane of rays, containing D*d* and *pn*, and cutting the face *aon*C in the line *an*. Hence *d′* is the shadow of *d*, and *ad′*, that of *ad*.

Similar constructions apply to the isometric projections and drawings of all rectangular bodies.

EXAMPLES.—1*st*. Find the shadow of the cube on the plane of its lower base.

2*d*. Find the shadow of *a*L (considered as C*a* produced) upon the rear face D*ao*.

3*d*. Find the shadow of a line parallel in space to C*a*, upon the face C*aon*.

4th. Find the shadow of Dq produced, on the plane of the lower base of the cube.

209. PROB. 3. *To construct the isometrical drawing of a carpenter's oil-stone box.* Pl. XIII., Fig. 103. This problem involves the finding of points which are in the planes of the isometric axes.

Let the box containing the stone be 10 inches long, 4 inches wide, and 3 inches high, and let it be drawn on a scale of $\frac{1}{5}$.

Assume C then, and make Ca=4 inches, Cb=10 inches, and Cc=3½ inches, and by other lines aD, Db, &c., equal and parallel to these, complete the outline of the box.

Represent the joint between the cover and the box as being 1 inch below the top, aCb. Do this by making Cp=1 inch, and through p drawing the isometric lines which represent the joint.

Suppose, now, a piece of ivory 5 inches long and 1 inch wide to be inlaid in the longer side of the box. Bisect the lower edge at d, make de=1 inch, and ee'=1 inch. Through e and e', draw the top and bottom lines of the ivory, making them 2½ inches long on each side of de', as at $e't$, $e't'$. Draw the vertical lines at t and t', which will complete the ivory.

210. To show another way of representing a similar inlaid piece, let us suppose one to be in the top of the cover, 5 inches long and 1½ inches wide. Draw the diagonals ab and CD, and through their intersection, o, draw isometric lines; lay off oo'= 2½ inches, and oo''=$\frac{3}{4}$ of an inch, and lay off equal distances in the opposite directions on these centre lines.

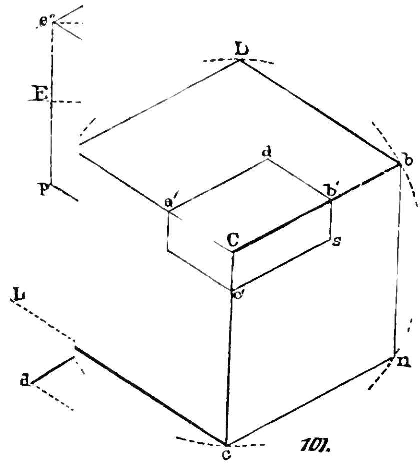
Through o', o'', &c., draw isometric lines to complete the representation of the inlaid piece.

Pl. XII., Fig. B, suggests other exercises, and illustrates the gaining of room by making the longest lines of the figure horizontal, as is often done in practice, when the longest lines of the original object are horizontal, since it is not the *position*, but the *form* of the figure which makes it isometrical.

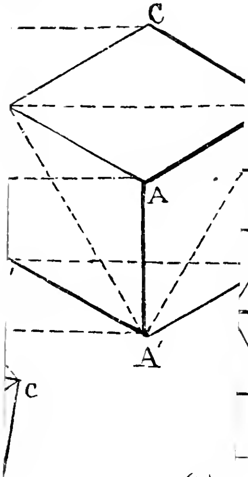
EXAMPLES.—*1st.* Reconstruct, isometrically, two courses of any of the examples in brick-work on Pl. V.

2d. Do likewise with any of the figures (46–49) of Pl. VII., assuming any convenient width for the timbers in each case.

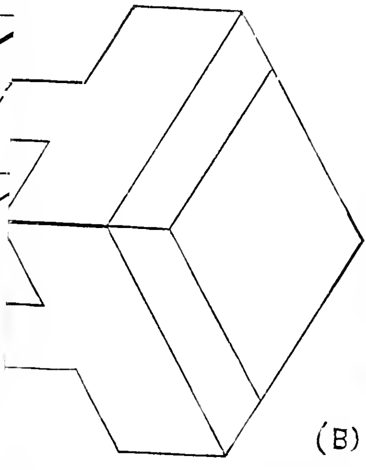
99.



101.

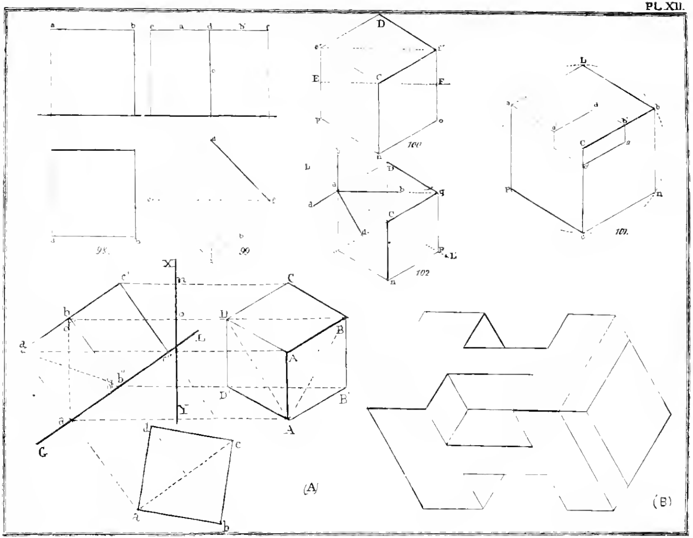


(A)



(B)

PL. XII.



211. PROB. 4. *To represent the same box (Prob. 3) with the cover removed.* Pl. XIII., Fig. 104.

This problem involves the finding of the positions of points not in the given isometric planes.

Supposing the edges of the box to be indicated by the same letters as are seen in Fig. 103, and supposing the body of the box to be drawn, 10 inches long, 4 inches wide, and $2\frac{1}{2}$ inches high; then, to find the nearest upper corner of the oil stone, lay off on Cb $1\frac{1}{2}$ inches, and through the point f, thus found, draw a line, ff'', parallel to Ca. On ff'', lay off 1 inch at each end, and from the points hh', thus found, erect perpendiculars, as hn, each $\frac{3}{4}$ of an inch long. Make the further end, x, of the oil stone $1\frac{1}{2}$ inches from the further end of the box, and then complete the oil stone as shown. To find the panel in the side of the box, lay off 2 inches from each end of the box, on its lower edge; at the points thus found, erect perpendiculars, of half an inch in length, to the lower corners, as p, of the panel; make the panels $1\frac{1}{2}$ inches wide, as at pp', and $\frac{1}{2}$ an inch deep, as at pr, which last line is parallel to Ca; and, with the isometric lines through r, p, p', and t, completes the panel.

212. *The manner of shading this figure* will now be explained.

The top of the box and stone is lightest. Their ends are a trifle darker, since they receive less of the light which is diffused through the atmosphere. The shadow of the oil stone on the top of the box is much darker than the surfaces just mentioned. The shadow of the foremost vertical edge of the stone is found in precisely the same way as was the shadow of the wire upon the top of the cube, Pl. XII., Fig. 102. The sides of the oil stone and of the box, which are in the dark, are a little darker than the shadow, and all the surfaces of the panel are of equal darkness and a trifle darker than the other dark surfaces. In order to distinguish the separate faces of the panel, when they are of the same darkness, leave their edges very light. The little light which those edges receive is mostly perpendicular to them, regarding them as rounded and polished by use. These light lines are left by tinting each surface of the panel, separately, with a small brush, leaving the blank edges, which may, if necessary, be afterwards made perfectly straight by inking them with a

light tint. The upper and left-hand edges of the panel, and all the lines corresponding to those which are heavy in the previous figures, may be ruled with a dark tint. In the absence of an engraved copy, the figures will indicate tolerably the relative darkness of the different surfaces; 1 being the lightest, and the numbers not being consecutive, so that they may assist in denoting *relative* differences of tint. When this figure is thus shaded, its edges should not be inked with ruled lines in black ink, but should be inked with pale ink.

Examples.—*1st.* Reconstruct, isometrically, any of the figures 53, 54, 55, 58, on Pl. VII.; or figures 63, 64, 65, 67, of Pl. VIII., omitting the bolts. [The few non-isometric lines that occur, being in given isometric planes, can be located by a litle care.]

2d. Construct the isometrical drawing of a low stout square-legged bench, and add the shadows.

3d. Construct the isometrical drawing of a cattle yard with pens of various sizes and heights.

4th. Construct the isometrical drawing of a shallow partitioned drawer, or box, with the shadows.

5th. Make the isometrical drawing of a cellar having a wall of irregular outline, and showing the chimneys, partitions, bins, etc.

CHAPTER III.

PROBLEMS INVOLVING NON-ISOMETRICAL LINES.

213. SINCE non-isometrical lines do not appear in their true size, each point in any of them, when it is in an isometric plane, must be located *by two isometric lines*, which, on the object itself, are *at right angles to each other*. Points, not in any known isometrical plane must be located by *three such co-ordinates*, as they are called, from a known point.

214. PROB. 5. *To construct the isometrical drawing of the scarfed splice, shown at* Pl. VIII., Fig. 66. Let the scale be $\frac{1}{16}$, or three-fourths of an inch to a foot. In this case, Pl. XIII., Fig. 105, it will be necessary to reconstruct a portion of the elevation to the new scale (see Pl. XIII., Fig. 106), where $Ap=1\frac{1}{2}$ feet, $pA''=6$ inches, and the proportions and arrangement of parts are like Pl. VIII., Fig. 66. In Fig. 105, draw AD, 3 feet; make DB and AA′ each one foot, and draw through A′ and B isometric lines parallel to AD Join A—B, and from A lay off distances to 1, 2, 3, 4, equal to the corresponding distances on Fig. 106. Also lay off, on BK, the same distances from B, and at the points thus found on the edges of the timber, draw vertical isometric lines equal in length to those which locate the corners of the key in Fig. 106. Notice that opposite sides of the keys are parallel, and that AV, and its parallel at B, are both parallel to those sides of the keys which are in space perpendicular to AB. To represent the obtuse end of the upper timber of the splice, bisect AA′, and make $va=Aa$, Fig. 106, and draw Aa and $A'a$. Locate m by va produced, as at ar, Fig. 106, and a short perpendicular$=rm$, Fig. 106, and draw mV and mV′; VV′ being parallel to AA′, and am being parallel to AV. To represent the washer, nut, and bolt, draw a centre line, vv', and at t, the middle point of Vn, draw the isometric lines tu and ue, which will give e, the centre of the bolt hole or of the bottom of the washer. A point —coinciding in the drawing with the upper front corner of the nut— is the centre of the top of the washer, which may be made $\frac{3}{8}$ of an inch thick.

Through the above point draw isometric lines, rr' and pp', and lay off on them, from the same point, the radius of the washer, say $2\frac{1}{2}$ inches, giving four points, as *o*, through which, if an isometric square be drawn, the top circle of the washer can be sketched in it, being tangent to the sides of this square at the points, as *o*, and elliptical (oval) in shape. The bottom circle of the washer is seen throughout a semi-circumference, i. e. till limited by vertical tangents to the upper curve.

On the same centre lines, lay off from their intersection, the half side of the nut, $1\frac{1}{4}$ inches, and from the three corners which will be visible when the nut is drawn, lay off on vertical lines its thickness, $1\frac{1}{4}$ inches, giving the upper corners, of which *c* is one. So much being done, the nut is easily finished, and the little fragment of bolt projecting through it can be sketched in.

EXAMPLE.—*Construction of the nut when set obliquely.* The nut is here constructed in the simplest position, i. e. with its sides in the direction of isometric lines. If it had been determined to construct it in any oblique position, it would have been necessary to have constructed a portion of the plan of the timber with a plan of the nut—then to have circumscribed the plan of the nut by a square, parallel to the sides of the timber—then to have located the corners of the nut in the sides of the isometric drawing of the circumscribed square. Let the student draw the other nut on a large scale and in some such irregular position. See Fig. 107, where the upper figure is the isometrical drawing of a square, as the top of a nut; this nut having its sides oblique to the edges of the timber, which are supposed to be parallel to *ca*.

215. PROB. 6. *To make an isometrical drawing of an oblique timber framed into a horizontal one.* Pl. XIII., Fig. 108. Let the original be a model in which the horizontal piece is one inch square, and let the scale be $\frac{1}{4}$.

Make *ag* and *ah*, each one inch, complete the isometric end of the horizontal piece, and draw *ad*, *hn* and *gk*. Let $ab = 2$ inches, $bc = 2\frac{1}{8}$ inches, and draw *bm*. Let the slant of the oblique timber be such that if $cd = 2$ inches, *de* shall be $\frac{3}{4}$ of an inch. Then by (213) *de*, parallel to *ag*, will give *ce*, which does not show its true size. Draw edges parallel to *ce* through *b* and *m*. In like manner *f* can be found, by distances taken from a side elevation, like Pl. VIII., Fig. 59.

216. PROB. 7. *To make an isometrical drawing of a pyramid standing upon a recessed pedestal.* Pl. XIII., Fig. 109. (From a

Model.) Let the scale be $\frac{1}{2}$. Assuming C, construct the isometric square, CABD, of which each side is $4\frac{1}{4}$ inches. From each of its corners lay off on each adjacent side $1\frac{1}{4}$ inches, giving points, as *e* and *f*; from all of these points lay off, on isometric lines, distances of $\frac{3}{4}$ of an inch, giving points, as *g*, *k*, and *h*. From all these points now found in the upper surface, through which vertical lines can be seen, draw such lines, and make each of them one inch in length, and join their lower extremities by lines parallel to the edges of the top surface.

Isometric lines through *g*, *m*, &c., give, by their intersections, the corners of the base of the pyramid, that being in accordance with the construction of this model, and the intersection of AD and CD is *o*, the centre of that base. The height of the pyramid—$1\frac{5}{8}$ inches—is laid off at *ov*. Join *v* with the corners of the base, and the construction will be complete.

217. *To find the shadows on this model.*—According to principles enunciated in DIVISION III., the shadow of *fh* begins at *h*, and will be limited by the line *hs*, *s* being the shadow of *f*, and the intersection of the ray *fs* with *hs*. *st* is the shadow of *f*F. According to (208), *w* is the shadow of *v*, and joining *w* with *p* and *q*, the opposite corners of the base, gives the boundary of the shadow on the pedestal. *ka* is the boundary of the shadow of *gk* on the face *ku*. The heavy lines are as seen in the figure. If this drawing is to be shaded, the numerals will indicate the darkness of the tint for the several surfaces—1 being the lightest.

218. PROB. 8. *To construct the isometrical drawing of a wall in batter, with counterforts, and the shadows on the wall.* Pl. XIII., Fig. 110. A wall in batter is a wall whose face is a little inclined to a vertical plane through its lower edge, or through any horizontal line in its face. Counterforts, or buttresses, are projecting parts attached to the wall in order to strengthen it. Let the scale be *one fourth*, i.e., let each one of the larger spaces on the scale marked 40, on the ivory scale, be taken as an inch.

Assume C, and make CD=2 inches and CI=10 inches in the righ and left hand isometrical directions. Make DF=6 inches, EF=$1\frac{1}{2}$ inches, FG=$1\frac{1}{2}$ inches, and GH=$1\frac{1}{2}$ inches, and join H and E, all of these being isometrical lines. Next draw EC, then as the top of the counterfort is one inch, vertically, below the top of the wall, while EC is not a vertical line, make F*d*=one inch, draw *de* parallel to FE and *ef*, parallel to EH. Make *ef* and C*a* each one inch, at *f* make the isometric line *fg*=$\frac{3}{4}$ of an inch, and at *a* make *ab*=2

inches, and draw bg. Make $bc=1\frac{1}{2}$ inches, draw ch parallel to bg and complete the top of the counterfort. The other counterfort is similar in shape and similarly situated, i. e., its furthermost lower corner, k, is one inch from I on the line IC, while HI is equal and parallel to CE.

219. *To find the shadows of the counterforts on the face of the wall.*—We have seen— Pl. IX., Fig. 78—that when a line is perpendicular to a vertical plane, its shadow on that plane is in the direction of the projection of the light upon the same plane, and from Pl. XII., Fig. 102, that the projection, an, of a ray on the left hand vertical face of a cube makes on the isometrical drawing an angle of 60° with the horizontal line mn'.

Now to find the shadow of mn. The front face of the wall— Pl. XIII., Fig. 110—not being vertical, drop a perpendicular, fo, from an upper back corner of one of the counterforts, upon the edge ba, produced, of its base, and through the point o, thus found, draw a line parallel to CI. $nfop$ is then a vertical plane, pierced by mn, an edge of the further counterfort which casts a shadow; and np is the direction of the shadow of mn on this vertical plane. The shadow of mn on the plane of the lower base of the wall is of course parallel to mn, and p is one point of this shadow, hence pq is the direction of this shadow. Now n, where mn pierces the actual face of the wall, is one point of its shadow on that face, and q, where its shadow on the horizontal plane pierces the same face, is another point, hence nq is the general direction of the shadow of mn on the front of the wall, and the actual extent of this shadow is nr, r being where the ray mr pierces the front of the wall.

From r, the real shadow is cast by the edge, mu, of the counterfort. r, the shadow of m, is one point of this shadow, and s, where um produced, meets yn produced, is another (in the shadow produced), since s is, by this construction, the point where um, the line casting the shadow, pierces the surface receiving the shadow. Hence draw srt, and nrt is the complete boundary of the shadow sought. The shadow of the hither counterfort is similar, so far as it falls on the face of the wall.

Other methods of constructing this shadow may be devised by the student. Let t be found by means of an auxiliary shadow of um on the plane of the base of the wall.

Remark.—In case an object has but few isometrical lines, it is most convenient to inscribe it in a right prism, so that as many of its edges, as possible, shall lie in the faces of the prism.

CHAPTER IV.

PROBLEMS INVOLVING THE CONSTRUCTION AND EQUAL DIVISION OF CIRCLES IN ISOMETRICAL DRAWING.

220. PROB. 9. *To make an exact construction of the isometrical drawing of a circle.* Pl. XIII., Figs. 111–112. This construction is only a special application of the general problem requiring the construction of points in the isometric planes.

Let Pl. XIII., Fig. 111, be a square by which a circle is circumscribed. The rhombus—Fig. 112—is the isometrical drawing of the same square, CA being equal to C′A′. The diameters $g'h'$ and $e'f'$ are those which are shown in their real size at gh and ef, giving g, h, e, and f as four points of the isometrical drawing of the circle. In Fig. 111, draw $b'a'$, from the intersection, b', of the circle with A′D′ and parallel to C′A′. As a line equal to $b'a'$, and a distance equal to $A'a'$ can be found at each corner of Fig. 111, lay off each way from each corner of Fig. 112, a distance, as Aa, equal to $A'a'$, and draw a line ab parallel to CA and note the point b, where it meets AD. Similarly the points n, o, and r may be found. Having now eight points of the ellipse which will be the isometrical drawing of the circle, and knowing as further guides, that the curve is tangent to the circumscribing rhombus at g, h, e and f, and perpendicular to its axes at b, n, o and r, this ellipse can be sketched in by hand, or by an irregular curve.

221. If, on account of the size of the figure, more points are desirable they can readily be found. Thus; on any side of Fig. 111, take a distance as $C'c'$ and $c'd'$ perpendicular to it, and meeting the circle at d'. In Fig. 112, make $Cc=C'c'$, and make cd equal to $c'd'$ and parallel to CD, then will d be the isometrical position of the point d'.

222. PROB. 10. *To make an approximate construction of the isometrical drawing of a circle.* Pl. XIII., Fig. 113. By trial we shall find that an arc, gf, having C for a centre and Cf for its radius, will very nearly pass through n; likewise that an arc eh, with B for a centre, will very nearly pass through r. These arcs will be tangents to the

sides of the circumscribing square at their middle points, as they should be, since Cf and Be are perpendicular to these sides at their middle points. Now in order that the small arcs, fbh and goe, should be both tangent to the former arcs and to the lines of the square at g, h, f and e, their centres must be in the radii of the larger arcs, hence at their intersections p and q. Arcs having p and q for centres, and ph and qe as radii, will complete a four-centred curve which will be a sufficiently near approximation to the isometrical ellipse, when the figure is not very large, or when the object for which it is drawn does not require it to be very exact.

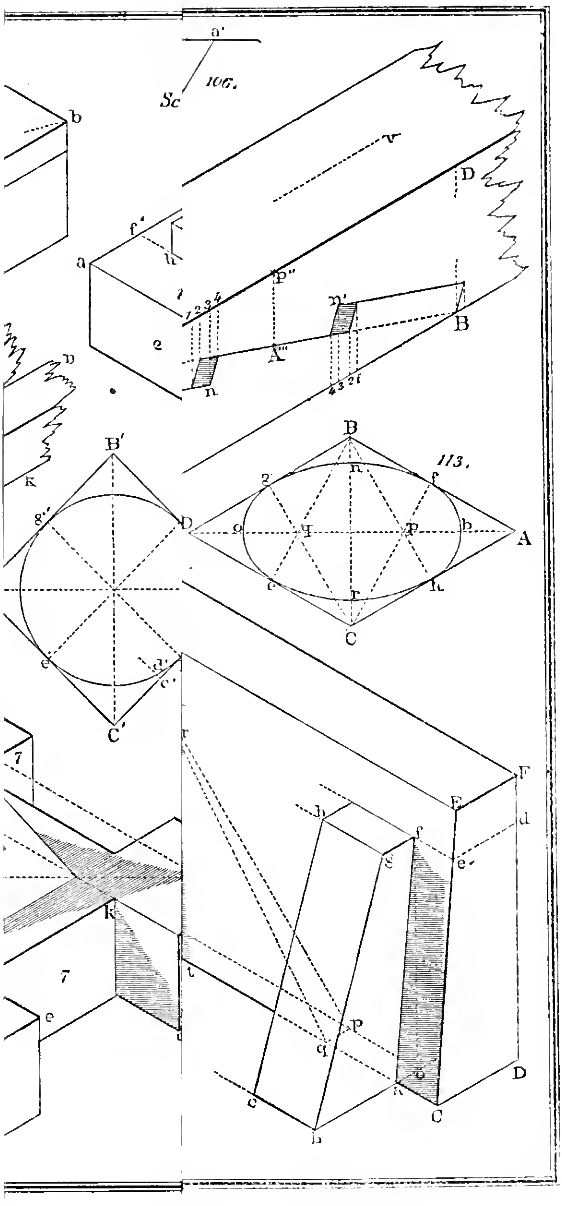
223. PROB. 11. *To make an isometrical drawing of a solid composed of a short cylinder capped by a hemisphere.* Pl. XIII., Fig. 114. Scale $=\frac{1}{4}$. Let this body be placed with its circular base lowermost, as shown in the figure. Make ac and bd the height of the cylindrical part $=1\frac{5}{12}$ inches, and draw cd. Now a sphere, however looked at, must appear as a sphere, hence take e, the middle point of cd, as a centre, and ec as a radius, and describe the semicircle ehd, which will complete the figure.

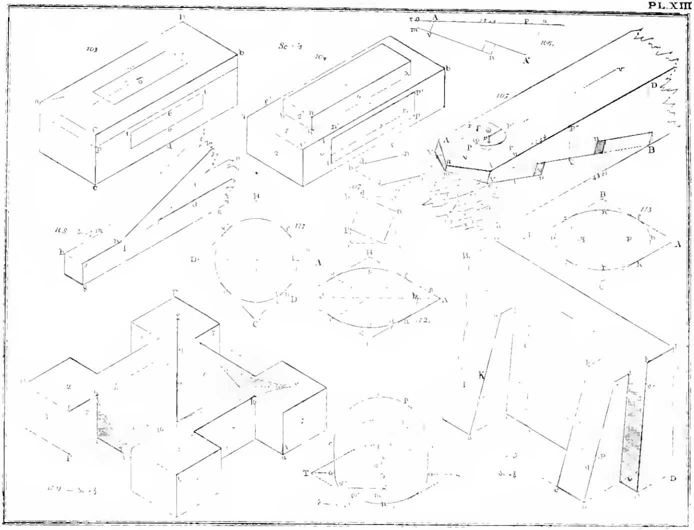
224. In respect to execution, in general, of the problems of this DIVISION, a description of it is not formally distinguished from that of their construction, since the figures generally explain themselves in this respect. In the present instance, the visible portion of the only heavy line required will be the arc anb. As there is no angle at the union of the hemisphere with the cylinder—see the preceding problem—no full line should be shown there, but a dotted curve parallel to the base and passing through c and d, might be added to show the precise limits of the cylindrical part.

225. Again, if it be desired to shade this body, the element, ny, of the cylindrical part, with the curve of shade, $pfry$, on the spherical part will constitute the darkest line of the shading. The curve of shade, $pfry$, is found approximately as follows. The line ny being the foremost element of the cylinder, $yy'=ne$, is the projection of an actual diameter of the hemisphere. mm'' and gg'', parallel to ST, are the radii of small semicircles of the hemisphere, to which projections of rays of light may be drawn tangent, and m' and g', are the true positions of their centres—ym' being equal to nm, and yg' equal to ng. Drawing arcs of such semicircles, and drawing rays, fd and rS, tangent to them, we determine f and r, points of the curve of shade on the spherical part of the body, through which, with p and y, the curve may be sketched.

226. PROB. 12. *To construct the isometrical circles on the three*

PL.XIII





visible faces of a cube, as seen in an isometrical drawing. Pl XIV., Fig. 115. This figure needs no minute description here, being given to enable the student to become familiar with the position of isometrical circles in the three isometrical planes, and with the positions of the centres used in the approximate construction of those circles. By inspection of the figure, the following general principle may be deduced. The centres of the larger arcs are always in the obtuse angles of the rhombuses which represent the sides of the cube, and the centres of the smaller arcs are at the intersection of the radii of the larger arcs with the diagonals joining the acute angles of the same rhombuses—i. e. the longer diagonals.

227. PROB. 13. *To make the isometrical drawing of a bird house.* Pl.XIV.,Fig.116. Assuming C, make CA′=16 inches, Ca=3 inches, and CB=9 inches. At a, make ab=1 inch, and ac=8 inches. Draw next the isometric lines BD and cD. Through b make bE=16 inches, make EA=3 inches, and EF=7 inches. Then draw the isometric lines DH and FH. Bisect bE at N, make Nf=11½ inches, draw cf and Ff, make ce=Fh=fg=one inch, and draw eg and hg. Through A and b draw isometric lines which will meet, as at a'. On bE make bk, lm and nE each equal to 3 inches, and let kl and mn each be 3½ inches. At l and n draw lines, as lv, parallel to CB, and one inch long, and at their inner extremities erect perpendiculars, each 3½ inches long. Also at k, l, m and n, draw vertical isometrical lines, as kt, 3½ inches long. The rectangular openings thus formed are to be completed with semicircles whose real radius is 1¾ inches, hence produce the lines, as kt—on both windows—making lines, as kG, 5¼ inches long, and join their upper extremities as at GI. The horizontal lines, as ts, give a centre, as s, for a larger arc, as tu. The intersection of Go with Iz—see the same letters on Fig. 115—gives the centre, p, of the small arc, uo. The same operations on both openings make their front edges complete. Make oq and pr parallel, and each, one inch long, and r will be the centre of a small arc from q which forms the visible part of the inner edge of the window. Suppose the corners of the platform to be rounded by quadrants whose real radius is 1½ inches. The lines $a'b$ and bk each being 3 inches, k is the centre for the arc which represents the isometric drawing of this quadrant, whose real centre on the object, is indicated on the drawing at y. So, near A, w is the centre used in drawing an arc, which represents a quadrant whose centre is x.—See the same letters on Fig. 115.

Of the Isometrical Drawing of Circles which are divided in Equal Parts.

228. Prob. 14. Pl. XIV., Fig. 117. *First method.*—If the semi ellipse, ADB, be revolved up into a vertical position about AB as an axis, it will appear as a semicircle AD′B of which ADB is the isometrical projection. Since AB, the axis, is parallel to the vertical plane, the arc in which any point, as D, revolves, is in a plane perpendicular to the vertical plane, and is therefore projected in a straight line DD′. Hence to divide the semi-ellipse ADB into parts corresponding to the parts of the circle which it represents, divide AD′B into the required number of equal parts, and through the points thus found, draw lines parallel to D′D, and they will divide ADB in the manner required. The opposite half of the curve can of course be divided in a similar manner.

229. *Second method.*—CE is the true diameter of the circle of which ADB is the isometrical drawing. Let it also represent the side of the square in which the original circle to be drawn is inscribed. The centre of this circle is in the centre of the square, hence at O, found by making *e*O equal to half of CE, and perpendicular to that line at its middle point *e*.

With O as a centre, draw a quarter circle, limited by CO and EO, and divide it into the required number of parts. Through the points of division, draw radii and produce them till they meet CE. CE, considered as the side of the isometrical drawing of the square, is the drawing of the original side CE of the square itself with all its points 1, 2,.........6, 7, &c., and O′ is the isometrical position of O. Hence connect the points on CE with the point O′ and the lines thus made will divide the quadrant BC′ in the manner required.

Applications of the preceding Problem.

230. Prob. 15. *To make an isometric drawing of a segment of an Ionic Column.* Pl. XIV., Fig. 118. Let *a*D be a side of the circumscribing prism of the column. By the second method of Prob. 14, find O′, the centre of a section of the column, and with O′ as a centre, draw any arc, as *a′q′*. The curved recesses in the surface of a column are called flutes, or the column is said to be fluted. In an Ionic, and in some other styles of columns, the flutings are semicircular with narrow flat, or strictly, cylindrical surfaces, as *ee″p*, between them. Hence, in Fig. 118, assume *a′b′*, equal to *q′v′*, as half of a space between two flutes, divide *b′v′* into four equal parts, and make the points of division central points of the spaces as *f′e′*

between the flutes. Let the flutes be drawn with points, as c' as centres and touching the points as $b'd'$; then draw an arc tangent, as at r, to the flutes. To proceed now with the isometrical drawing, draw, in the usual way, the isometrical drawing of the outer circumferences of the column, tangent to aD and b'''F—assuming DF for the thickness of the segment. Now $a'q'$ being any arc, and not one tangent to aD so as to represent the true size of a quadrant of the outer circumference, the true radius of the circle tangent to the inner points of all the flutes will be a fourth proportional, O$'y'$ to O$'f'$, Oi (=O$'y$), and O$'s$. On Oi, lay off OY=Oy', draw IJ to find a centre I, and similarly find the other centres of the larger arcs of the inner ellipse. The points n, h and n', h' are the centres of the small arcs (222) for the two bases. Having gone thus far, produce O$'b'$, O$'c'$, &c. to aD; at b, c, &c., erect vertical lines, bb''', cc''', &c., then from b, c, &c. draw lines to O, and note their intersections, b'', c'', &c. with the curves of the lower base; and from b''', c''', &c. draw lines to O$''$ and note their intersections, b'''', c'''', &c. with the ellipses of the upper base. This process gives three points for each flute by which they can be accurately sketched in, remembering that they are tangent to the inner dotted ellipses, as at c'''', o''', &c. and to the radii, as e''O$''$—at e''. Parts beyond FO$''$ are projected over from the parts this side, thus drawn.

231. PROB. 16. *To construct the isometrical drawing of a segment of a Doric Column.* Pl. XIV., Fig. 119. The flutes of a Doric column are shallow and have no flat space between them. Adopting the first method of Prob. 14, let the centre, A, of the plan be in the vertical axis, GA$'$, of the elevation, produced. Let Ac and Ab be the outer and inner radii containing points of the flutes. Make Ad=$\frac{4}{3}$ of Ac, for the radius of the circle which shall contain the centres of the flute arcs. Let there be four flutes in the quadrant, shown in the plan. Their centres will be at h, &c., where radii Ag, &c., bisecting the flutes, meet the outermost arc. In proceeding to construct the isometrical drawing, project b and c, at b and c' on the axis A$'d'$. Now, owing to the variation at b and c' between the true and the approximate ellipse, we cannot make use of the latter, if we retain b' and c' in their proper places, as projected from b and c, hence through b' and c' draw isometric lines which locate the points N$'$ and Q$'$ (the points are between these letters) which are the true positions of N and Q respectively. Corresponding points, between N$'''$ and v, are similarly found. By an irregular curve the semi-ellipses vb'Q$'$ and N$'''c'$N$'$ can be quite accurately

drawn. Next, project upon these curves the points u, e, &c., r, g, &c. of the flutes—as at u', e', &c., r', &c., and with an irregular curve draw the curves through these points, tangent to the inner semi-ellipse. The corresponding curves of the lower base are found by drawing lines $r'r''$, $u'u''$, &c. through the points of tangency, r', k', &c., and through u', &c., and all equal to FD, the thickness of the segment. The curves above the axis $A'd'$ are projected across from those already made below it. Let this figure and the last be made *separately on a very large scale.*

Special Examples.

232. PROB. 17. *To draw a cube or other parallelopipedical body so as to show its under side.* Pl. XIV., Fig. 120. By reflection, it becomes evident that it is the relative direction of the lines of the drawing among themselves, that make it an isometrical drawing. Hence in the figure, where all the lines are isometric lines, the whole is an isometric drawing, now that the solid angle C is nearest us, as much as if the angle A (lettered C on previous figures) were nearest us.

233. *Remark.* By a curious exercise of the will, we can make Fig. 120 appear as an interior view, showing a floor CFED, and two walls; or, in Fig. 115 and others, we can picture to ourselves an interior showing a ceiling GIk and two walls. This is probably because—1*st.* All drawings being of themselves only plane figures, we educate the eye to see in them, what the mind chooses to conceive of, as having three dimensions. 2*nd.* When, as in isometrical drawing, the drawing in itself as a plane figure, is the same for an interior as for an exterior view of any given magnitude, the eye sees in it whichever of these two the mind chooses to imagine.

234. PROB. 18. *To construct isometrical drawings of oblique sections of a right cylinder with a circular base.* Pl. XIV., Fig. 121. This construction is easily made from a given circle as a base of the cylinder, that base being in an isometric plane. The circle in the plane AGEF is such a circle. Let A′G′E′F′ be a plane inclined to AGEF but perpendicular, as the latter is, to the planes GB and DF, and let A″G″E″F″ be a plane inclined to all the sides of the prism AGE—D.

Lines, as $aa'a''$, &c., being in the faces of the prism and parallel to their edges, meet the intersections, F′E′—F″E″, &c. of the oblique planes at points a', a'', &c., which are points of oblique sections of a cylinder inscribed in the prism AGE—D, and whose base is $acbdu$.

So, points, as c, have the corresponding points $c'c''$, &c. in the diagonals A′E′, A″E″ of the planes in which those points are found.

To find points, as t', t'', &c. corresponding to t in the base, draw any line, as yd, through t, and find the corresponding lines, as $y'd'$ and $y''d''$. Their intersections with the diagonals G′F′ and G″F″ will give the points t', t'', &c. Having thus found eight points of each oblique section of the given inscribed cylinder whose base is $abcd$–u, and remembering that each of these sections is tangent to the sides of its circumscribing polygon (considering the lines $y'd'$,&c.), the curves a', b', c', t', and a'', b'', c'', t'' are readily sketched in.

235. *Remarks.* *a.* As before stated, it is the relative direction, among themselves, of the lines of an isometrical drawing, that determine it as an isometrical drawing, hence Pl. XIV., Fig. 121, is an isometrical drawing, though its lines are not situated with reference to the edges of the plate as the similar lines of previous figures have been. If the portion of the plate containing this figure were cut out so as to make the edges of the fragment, so cut out, parallel and perpendicular to GE, the figure would appear like the previous isometrical drawings.

b. The problem just solved must not be confounded with one which should seek *to find the isometric projection of a curve which in space is a circle on the plane* G′E′—A′, for the curve $a'b'c'd't'$ is not a circle, in space.

236. Prob. 19. *To solve the problem just enunciated.* Pl. XIV., Figs. 121–122. $e''r$—Fig. 122—is a plan of the section rA′F′ in which—it being a square—a circle can be inscribed. $e''r$ is therefore the plan of the circle also. Making rG—Fig. 122—equal to rG′—Fig. 121, and drawing e''G, we have the plan of the section G′E′—A′, and making $o''p'$, Fig. 122, equal to $e''r$, we have the plan of a circle in the section G′E′—A′. Now draw $o''x$ and $p'e'$—Fig. 122—make A′e and G′e''' and $e'''p$ and eo—Fig. 121—equal to $e''x$, Ge', $e'p'$ and xo''—Fig. 122—draw pY and oU; and u'U and b'Y to intersect them, and we shall have U and Y as the isometric positions in the plane G′E′—A′ of the points o' and p' which, considered as points on the circle, are evidently enough extremities of its horizontal diameter, at which points, the circle is tangent to the vertical lines whose isometric positions in the plane G′E′—A′ are pY and oU. T and a' are other points.

The finding of intermediate points, which is not difficult, is left as an exercise for the student.

CHAPTER V.

OBLIQUE PROJECTIONS.

1. THERE is a kind of projection, examples of which, in the drawing of details, etc., are oftener seen in French works than isometrical projection (an English invention) is. It has been variously named, "Military," "Cavalier," or "Mechanical" Perspective. It may be called "Cabinet Projection," it being especially applicable to objects no larger than those of cabinet work, and being actually used in representing such work. It is properly called *oblique projection*, because in it the projecting lines, which have been hitherto made perpendicular to the plane of projection are oblique to that plane; and *pictorial projection*, on account of its pictorial effect, as seen in Pl. I., Figs. 1, 2, 3, etc.

2. This new projection differs from isometrical, chiefly in showing *two* of the three dimensions of a cube, for example, in their true *direction* as well as *size*.

Thus; Fig. 1 is the isometrical drawing of a cube, and Fig. 2 is

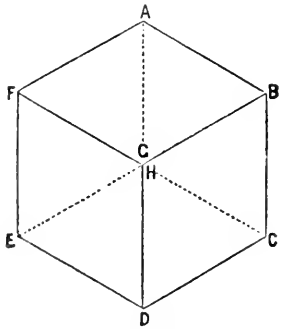


Fig. 1.

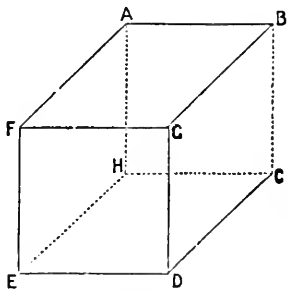


Fig. 2.

an oblique projection of the same cube; all the edges being of the same length in both figures. Hence we see, as stated, that in the latter figure, the faces DEFG, and ABCH, and by consequence every line in them, are shown in their true *form*, as well as ***size***; which is not true of isometrical drawing.

3. Another advantage of oblique projection, already apparent, is, that the remote corner, H, which, in the isometrical drawing of a cube, coincides with the foremost corner, G, is seen separately in the oblique projection.

Also, of the four body diagonals of the cube, one, GH, appears as a point only, in isometrical projection, and the other three, as FC, are all partly confounded with the projections, as FG, of edges. But in the oblique projection, all these diagonals show as lines, and, except BE, separately from the edges of the cube.

4. In the projections hitherto considered, the projecting lines of a point have uniformly been taken perpendicular to the planes of projection.

Sometimes, however, the projecting lines, or direction of vision (12) are oblique to the plane of projection.

There are thus two systems of projection in which the eye is at an infinite distance. *First*, common or *perpendicular projections*, in which the projecting lines (5) are perpendicular to the plane of projection. *Second. Oblique projections*, in which those lines are oblique to the plane of projection.

Isometrical, is a species of perpendicular projection. We shall now proceed to explain the simple and useful form of projection which is called oblique projection.

5. If a line, AB, Figs. 3, or 3*a*, be perpendicular to any plane PQ, its projection on that plane, in *common* projection, would be simply the *point* B. But if we suppose the projecting line, AC, of any point, A, to make an angle of 45° with the plane of projection PQ, it is evident that the projection of A would be at C, and the projection of AB on PQ would be BC; also that BC=AB. That is, the *projection*, as BC, *of a perpendicular to the plane of projection is equal to that perpendicular itself.*

Any line through A and *parallel* to the plane PQ would evidently be *projected in its real size on* PQ. Hence, finally, the system of oblique projections here described, allows us to show the *three dimensions* of a solid in *their real size*, on a single figure; but only *parallels*, and *perpendiculars* to the plane of projection, appear in their true size.

6. It is now evident from Fig. 3*a*, that there may be an infinite number of lines from A, each making an angle of 45° with the plane PQ. These lines, taken together, would form a right cone with a circular base, whose axis would be AB, whose vertex would be A, and whose base would be a circle in the plane PQ, drawn with B as a centre, and BC as a radius. Each radius of this circle

would be an oblique projection of AB, corresponding with the element as C′A, from its extremity, taken as the direction of the projecting lines. That is, *the oblique projection* of AB *may be drawn equal* to AB and in any direction.

7 Thus Figs. 4, 5, 6 and many more are all equally oblique projections of the same cube. The paper represents the plane of projection; FA is perpendicular to the paper at A. The eye, relative to Fig. 2, is looking *from an infinite distance* above and to the right of the body, and *in a direction* making an angle of 45° with the paper. And, generally, in oblique projection, the *direction of vision=the projecting lines*, may have any direction (the same for all points in the same problem) *making an angle of* 45° *with the plane of projection.* Hence FA may have any direction relative to FG and yet be always equal to FG, that is to the original of FA in space. Thus, CDK=45°, in Fig. 4; 30°, in Fig. 5; and 60°, in Fig. 6. Also, DC, FA, etc., may incline to the *left*, or *downward.*

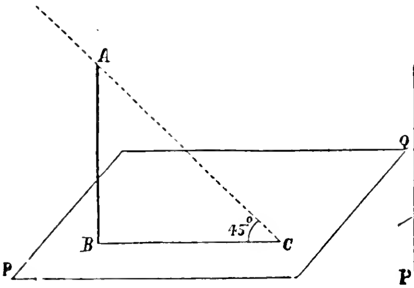


FIG. 3.

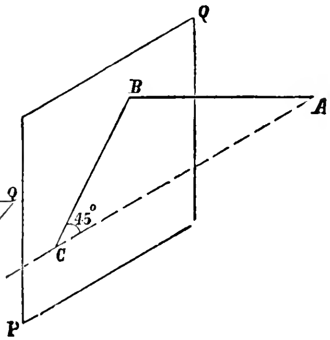


FIG. 3a.

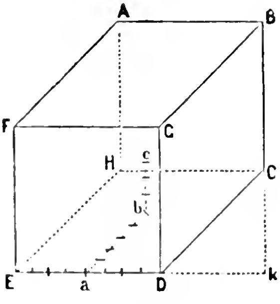


Fig. 4.

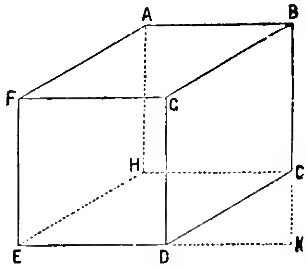


Fig. 5.

In Fig. 6, CDK=60°. Accordingly DK=$\frac{1}{2}$ DC, from which, having found K, the perpendicular KC can be drawn to limit DC. Or, as before, CK=$\frac{1}{2}\sqrt{3}$, DC being =1. That is, CK=$\frac{1}{2}$EC, since CDE=120°. And for a square prism of any length, KC= half of the diagonal joining alternate vertices of a regular hexagon whose side equals the edge of the prism, lying in the direction of DC, and whose length is supposed to be given.

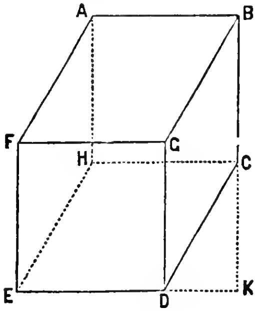


Fig. 6.

With these illustrations, the student might proceed to investigate other relations between the parts of these, and still other oblique projections. But the above may suffice for now.

We observe that, in Figs 5 and 6, none of the body diagonals are confounded with the edges; and that each of the three forms may be preferable for certain objects.

8. Points not on the axes EF, EH, and ED, or on parallels to them, are found by co-ordinates, as in isometrical drawing. Thus, if ED, Fig. 4, be 4 inches, and if we make Ea=2 inches, ab parallel to EH,=2$\frac{1}{2}$ inches, and bc, parallel to EF,=1$\frac{1}{2}$ inches, then c is the oblique projection of a point, 2 inches from the face FH; 2$\frac{1}{2}$ inches from the face FEG, and 1$\frac{1}{2}$ inches above the base EDC. This principle will enable the student to reconstruct any of the preceding isometrical examples of straight-edged objects, in oblique projection.

9. It only now remains to explain the oblique projections of circles. Let Fig. 7 be the oblique projection of a cube, with circles inscribed in its three visible faces. One of these circles, $abcd$, will appear as a circle, and so would the invisible one on the parallel rear face.

For the ellipse in BCDG, draw the diagonals, BD and CG, of

that face. Then in the cube itself, horizontal lines joining corresponding points in the circles *abcd*, and *hpfu*, are parallel to the diagonal EC. Hence *tu*, *ef*, *mp*, and *gh* determine the points *u*, *f*, *p*

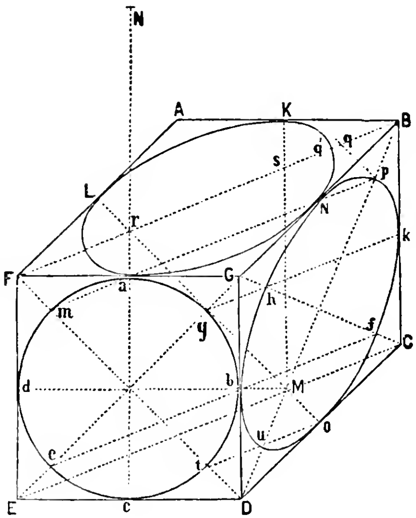


Fig. 7.

and *h*, by their intersections with the diagonals BD and CG. The middle points of the sides of the face BCDG, are also points of the ellipse, and are its points of contact with those sides. The ellipse also has tangents at *h* and *f*, parallel to BD, and at *u* and *p*, parallel to CG. Hence, having eight points, all of which are points of contact of known tangents, the ellipse can be accurately sketched.

10. The ellipse in the upper face could be found in the same manner. But an approximate construction by circular arcs has been shown, to test its accuracy and appearance, as compared with the approximate isometrical ellipse. The ellipse being tangent at L and K, perpendiculars to FA and BA, at those points, will intersect at M, the centre of an arc tangent to FA and BA at those points. Then *a*N perpendicular to FG at *a*, and equal to MK, gives N, the centre of the arc *an*. As the remaining arcs must be tangent to those just drawn, their centres, *r* and *s*, must be the intersections of the radii of the large arcs, with the transverse axis, FB, of the ellipse.

The true extremities of the transverse axis are found by drawing

pq parallel to AC, and a parallel to it from *u*. The error *qq'* at each end of the transverse axis, is thus seen to be considerable Also the greater difference between the radii, than occurs in making the isometrical ellipse, occasions a harsh change of curvature at K, N, etc.; so that the approximate construction of the oblique ellipse is of very little value.

11. It is found on trial, that the centre M falls both on BD and EC, so that neither KM nor LM really need be drawn. The reason of this property, which so simplifies the construction, is evident. For, BA = AF = FE are in position as three sides of a regular octagon, so that the perpendiculars, as KM, from the middle points of those sides, will meet at the same points with BD, AGM, EC, etc., which are obviously the bisecting lines of the angles of the octa gon, viz. at the centre, M, of the octagon.

By varying the angle GFA, as in the previous figures, the student may discover similar coincidences, which he can explain for himself.

Finally, it is to be noticed, that the pictorial diagrams of Pl. I., Figs. 1, 2, 3, 5, etc., which are so effective a substitute for actual models, to most eyes, are merely oblique projections of models themselves.

Practical Examples.

12. Pl. XV. shows some further illustrations of oblique projection in contrast with isometrical drawing.

Fig. 1 is an isometrical drawing of a roller and axle, showing the parallel circumscribing squares of its several parallel circles; and the circumscribing prism, *mnpo*, of the roller, placed so as to show its lower base. The distances *ab*, *bd* and *de*, between the centres of the circles, are thus seen in their true size, in this, and on the next two figures. Also the several circles and their centres, have the same letters on the same figures.

Fig. 2 shows an oblique projection of the same object, when its axis, *ae*, is made *perpendicular* to the paper. This is the simplest position to give to the object, since its several circles, being then *parallel* to the paper, will appear respectively as equal circles in the figure. And, generally, in making oblique projections of objects having some circular outlines, the object should be so placed, that the majority of these outlines should be in planes parallel to the plane of projection.

Example.—Make *ae* in any other direction.

Fig. 3 shows another oblique projection of the same object, but

with its axis *ae* parallel to the paper, or plane of projection. Different wheels and their axles in the same machine, might have the two positions indicated in Figs. 2 and 3. Hence it is necessary to understand both; though if drawing only a single object of this kind, we should for convenience make it as in Fig. 2, only remembering, as explained in previous principles, that *ae* may be drawn in any direction.

EXAMPLES.—1*st.* In Fig. 1, let the *upper* end of the axis be the visible one.

2*d.* In Fig. 3, let *ae* be *horizontal* and parallel to the paper and let the left hand end of the body be seen.

Pl. XV., Figs. 4, 5, and 6 are a plan and two isometrical drawings, in full size, of a hexagonal nut. Fig. 4 is the plan of the nut with the circumscribing *rectangle*, *mnop*, containing two of its sides, CD and AF. Fig. 5 is the isometrical drawing of the same, and thus shows the face CD*h*, in its true size. The edges B*b*, C*c*, etc., and centre heights, H*h*, of the faces, also show in their real size, as does the height O*o* of the nut. BC is greater than its real size, being more nearly parallel to *pn* than *pm* is. AB is less than its true size, AB, Fig. 4; being nearer perpendicular to *pn* than *pm* is.

Fig. 6 is, perhaps, a more agreeable looking isometrical figure of the nut, but it shows only the heights in their true sizes, except as *pq* equals the diameter of the circumscribing circle MNL of the hexagonal base of the nut, so that half of *pq* equals the true width of the faces.

This figure makes an application of (Prob. XIV., *First Method*). Thus, having made the isometrical circle MNKL, in the usual way, describe the semicircle MN′K′L on ML as a diameter, and inscribe the semi-hexagon in it with vertices as M, N′, K′ and L. Then by revolving the semicircle back to its isometrical position, N′ and K′ will fall at N and K.

The surface Q, in Figs. 5 and 6, represents a spherically rounded surface of the nut, while the surface, R, is plane. By finding three points as *c*, *h* and D, Fig. 5, in each upper edge of a face, those edges can be drawn as circular arcs; and the visible boundaries, *qq*, of the rounded surface Q can be sketched, as indicated.

EXAMPLES.—1*st.* Make *oblique projections* corresponding to Figs. 5 and 6.

2*d.* Also with the *top*, R, of the nut *parallel to the plane of projection*, and *either* in isometrical or oblique projection.

3*d.* Also as if Fig. 6 were turned 90° about O*o*, so as to show only *two* faces of the nut.

Figs. 7, 8, and 9 show a plan and oblique projection of a model of an oblique joint.

Fig. 7 shows, once for all, that in every case of *oblique*, as well as of *isometrical* drawing, where the lines as *dg* and *gp*, of the object, are oblique to each other, the body must be conceived to be inclosed in a circumscribing rectangular prism, whose sides shall contain its points, or from which they can be laid off by ordinates as *mo*, parallel or perpendicular to those sides.

Fig. 7 is on a scale of one half, and Fig. 9 is in full size. Then, supposing the scale to be the same C*n*, Fig. 9, = *cn* Fig. 7, *mo*, *n*P, *ih*, etc., in Fig. 9 = *mo*, *np*, *ih*, etc., in Fig. 7. So *fe* and *fs* Fig. 9 = the same in Fig. 7.

Thus the edges of timber A are shown in their real size, but those of B are distorted by their position. B is separately shown in its true proportions in Fig. 8, that is so far as its lines are parallel to *ko*, *o*O or *op*.

Of the heavy lines in Oblique Projection.

These simply follow the same rule, relative to the given object that is applied in common, or perpendicular projections; (16–20).

Thus, in Pl. XV., Fig. 2, the semicircles of A, B and C below *ae* would be heavy, and the opposite parts of D, and E. Also if B, Fig. 8, represents a timber parallel to the ground line, the heavy lines would be as there shown. And likewise on Fig. 9, where these lines are indicated by double dashes across them.

In short, conceive the common projections of an object to be given with the heavy lines drawn. The oblique projection of the same object, placed in the same position, would simply show the oblique projections of the same heavy lines. That is, the same lines would be heavy in both kinds of projection.

DIVISION FIFTH.

ELEMENTS OF MACHINES.

CHAPTER I.

PRINCIPLES. SUPPORTERS AND CRANK MOTIONS.

General Ideas.

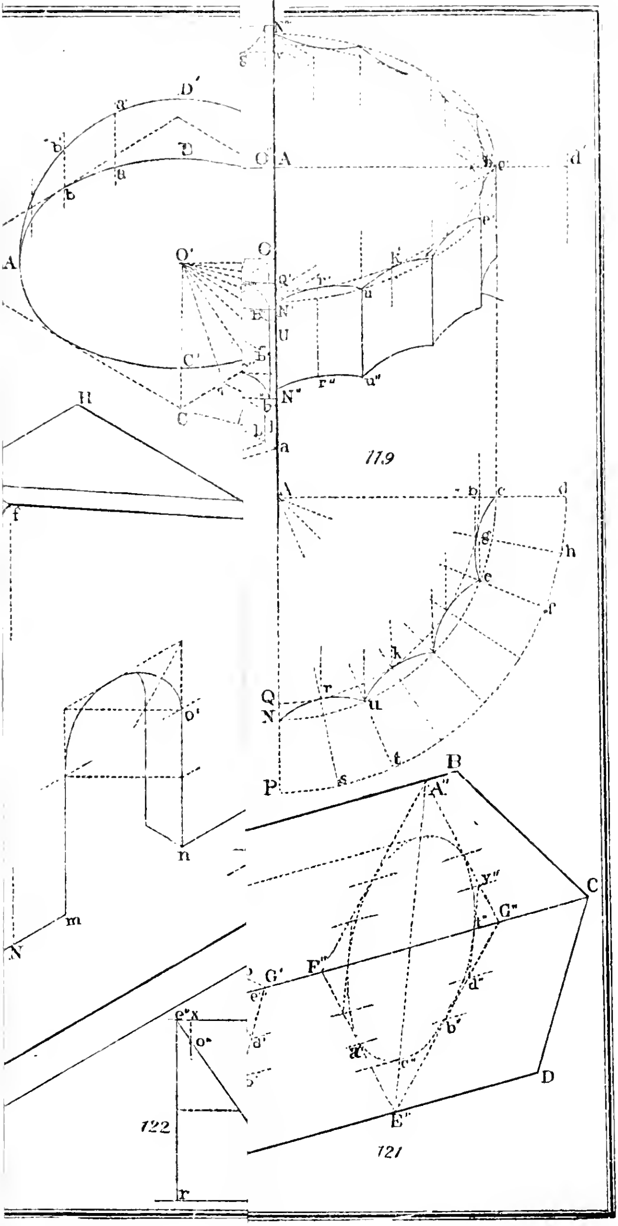
1. MACHINES generally effect only *physical* changes. That is, they are designed to change either the *form* or the *position* of matter. They do this either *directly*, as in machines that operate immediately on the raw material to be wrought, as looms, lathes, planers of wood or metal, etc., or *indirectly*, as in the machines called *prime movers*, like steam-engines and water-wheels, which actuate operating machines.

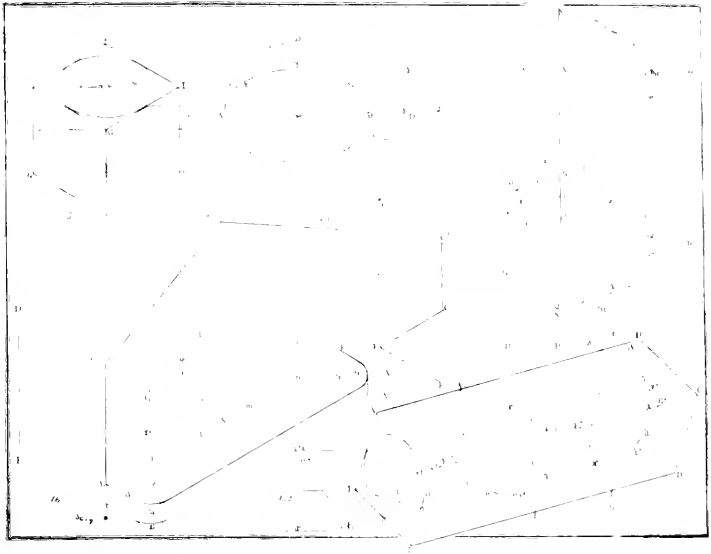
We have, then, *Prime movers* and *Operative machines*. Also, of the latter, machines for changing the *position* of matter, as pumps, cranes, etc.; and machines for changing its *form*, as lathes, planers, etc.; and each with many subdivisions.

2. In every machine there are to be distinguished the *supporting parts*, which are generally fixed and rigid, and the *working parts*, which are moving pieces.

The supporting parts are *general*, supporting the entire machine; or *local*, supporting some one part, as the *pillow-block*, also called a plummer-block, or a pedestal, which supports a revolving shaft; or the *guide bars*, plainly seen in some form at the piston-rod end of the cylinder of any locomotive or other steam-engine, and which, by means of the stout block, called a *cross-head*, sliding between them, constrain the piston-rod, which is fastened to the cross-head, to move in a straight line.

3. *The working parts are connected together*, forming a *train*, subject to this law, that *a given position of any one piece determines that of all the others*. For the purpose of making the drawing of a machine, it is not enough, therefore, only to take


119
121
122



the measurements of its parts. This will suffice for the frame, but the motions of the train must be understood, so as to know what *position* to give to other parts, corresponding to a given position of some one part.

4. In some machines, however, there are *subordinate trains*, serving to adjust the position or speed of the principal trains, as in case of engine governors. Also some parts are adjustable by hand, as the position of the bed in a drilling machine, or of the tool and rest containing it, in a lathe.

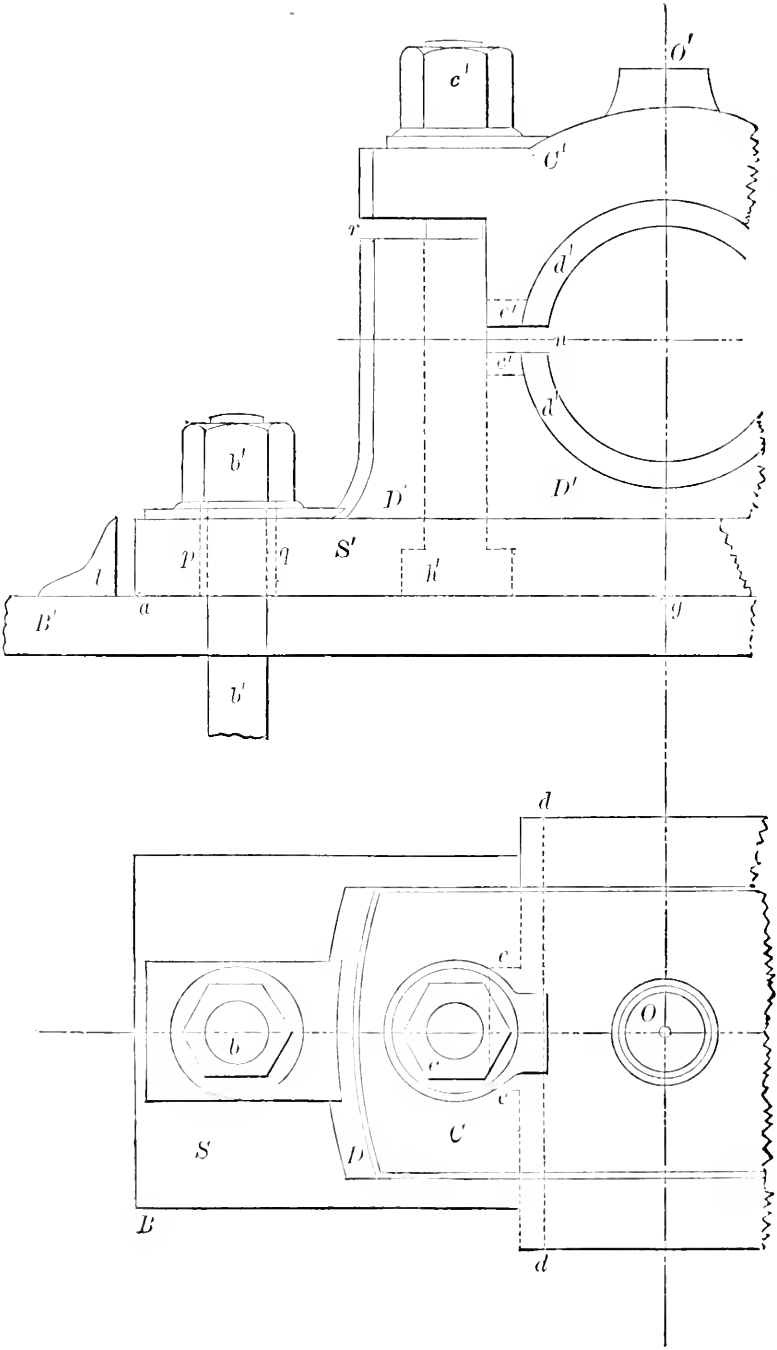
5. The working parts of every machine consist of certain mechanical elements or organs, which are comparatively few in number, and not always all present in any one machine. The principal of these are pistons, cross-heads, shafts, cranks, cams and eccentrics, toothed wheels, screws, band-pulleys, connecting-rods, bands or chains, sliding or lifting valves, grooved links, rocking arms and beams, flat or spiral springs, chambered parts and internal passages, as pump-barrels, steam-cylinders, valve-chests, etc.

6. These, considered separately, are of various degrees of complexity of design, many of them quite simple. By far the most important, relative to the geometrical theory of their perfect action, are toothed wheels of various forms. These we shall therefore principally consider, together with a few other useful examples.

Supporters.

EXAMPLE 1. **A Pillow-block.** Pillow-blocks of various designs, adapted to horizontal, or vertical, or beam engines, are so common, and so generally represented in works on practical mechanism, that the following figure, taken from a drawing to scale, is inserted here as a sufficient guide; the object, moreover, being symmetrical with respect to the centre line OO′, and a little more than half shown.

Description.—BB′, not definitely shown in plan, is a portion of the main bed of an engine. SS′ is the sole of the pillow-block, DD′ its body, CC′ its cover, and *dd*—*d′d′* the brasses which immediately enclose the fly-wheel shaft of the engine. The holding-down bolts, as *b*—*b′b′*, pass through slotted holes *pq*, a little wider, that is, in the direction *pq* than the diameter


O'
c'
C'
r
d'
e'
n
e'
d'
b'
D'
D'
S'
p
q
h'
l
B'
a
g
b'
d
c
O
b
c
c
C
S
D
B
d

of the bolt. This construction allows for adjustment of the position of the block by wedges, driven between the sole and stops l, solid with the bed B′.

The cover CC′ is held in place by bolts c—$c'h'$, the heads h' being in recesses, sunk in the under side of the sole. The spaces at r and n between the cover and the body, allow for the wear of the brasses dd—$d'd'$ against the shaft. To prevent lateral or rotary displacement of the brasses, ears ee—$e'e'$ project from them into recesses in the cover and body of the block. The same end is often attained by making their outer or convex surfaces octagonal, and by providing them with flanges where they enter and leave the block. OO′ is the oil cup, here solid with the cover, but oftener now a separate covered brass cup, contrived to supply oil gradually to the shaft.

Construction.—The *proportions* of the figure being correct, assume ag, the half length of the sole, to be 12 inches, and measure by a scale its actual length on the figure. A comparison of the two will indicate the corresponding scale of the figure.*

Then, having determined the scale, all the other measurements can be determined by it to agree with each other, and the figure can be *drawn* on any scale desired, from $\frac{1}{3}$ to $\frac{1}{6}$ of the full size.

The body being symmetrical, all the measurements to the left from OO′ can be laid off to the right of it, and the complete projections thus constructed.

The method of drawing hexagonal nuts has been shown in detail in Div. I., Problems 31, 32.

Execution.—Note the heavy or shade lines as in previous examples (Div. II.); but if the figure is to be shaded and tinted, ink it wholly in pale lines or none.

Exercises.—1. Construct from the two given projections an end elevation of the block.

2. Construct a vertical section on the centre line Ob.

3. Construct a top view with the cover removed. (The dotted lines showing the internal construction will enable these sectional views to be made.)

* The proportions adopted by different builders, and by the same builder for different cases, being not precisely alike, the pupil is thus encouraged not to think any one set of given measurements indispensable.

Ex. 2. **A Standard for a Lathe.** Pl. XVI., Fig. 1.

Description.—This example illustrates the application of tangent lines and circles to the designing of open frames, having outlines conveniently varied for use and economy of material.

The figure shows half of the side view (the object being symmetrical), also an edgewise view. The scale, $\frac{1}{8}$, being given, and the *operations* of construction being here more important than the precise measurements, only a few of the principal dimensions are given in this and in the following figures, leaving the rest to be assumed, or sufficiently determined by knowing the scale.

The double lines on the edges indicate ribbed edges, so made to secure stiffness and strength. The central and triangular openings may also afford rests for long-handled tools or metal bars. The nut *n* secures the standard to the lathe-bed.

Construction.—Having made the half widths $4\frac{1}{4}''$ and $12\frac{1}{2}''$ at top, and at AB, the outline BD may be drawn. This is composed of an arc of 60° with radius AB, a tangent to this, and a second arc of 60° tangent to the last line, and with its centre on a horizontal line through D.

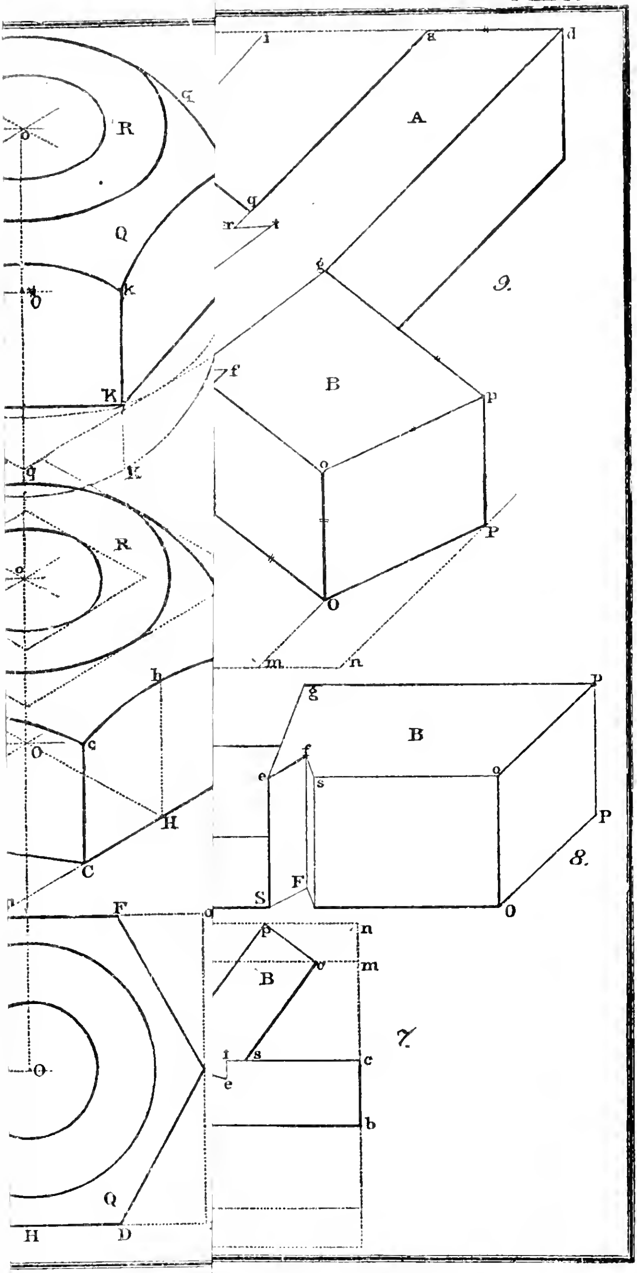
The outline of the central opening is partly concentric with the arc through D, partly circular with C as a centre, and partly circular as shown at the top, and there tangent to the side arcs. C is here taken on a horizontal line through the lower end of the arc from D.

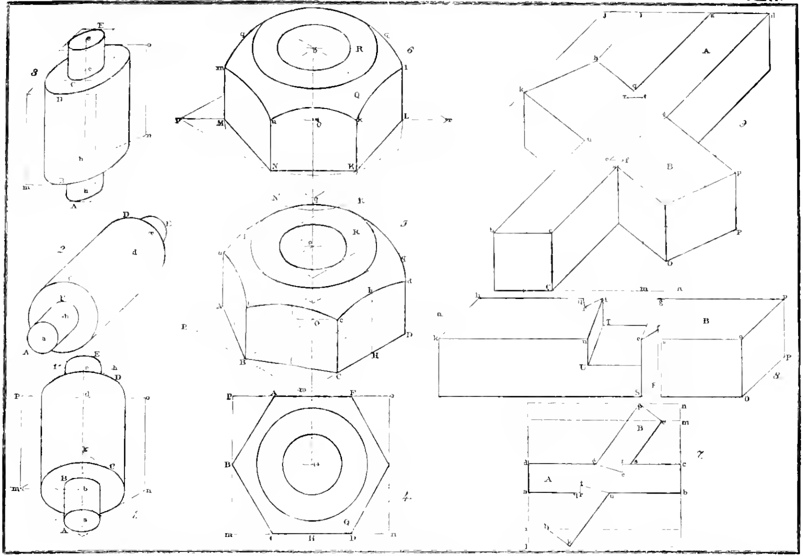
Execution.—The figure, both halves of which should be drawn, and on a little larger scale, as $\frac{1}{5}$ or $\frac{1}{6}$, gives occasion for the neat drawing of curved shade lines, and the neat connection of tangent outlines.

Exercise.—1. Vary the design by making the arcs from B and D each less than 60°, and so that C shall be on the radius through the lower limit of the arc through D.

Ex. 3. **Section of an Engine-bed, Guides, and their Support.** Pl. XVI., Fig. 2.

Description.—ABCD is a cross-section of the bed of a horizontal engine, which is of uniform section throughout. *hh* is a vertical plate, bolted to the bed as shown at *n*. From this plate, and solid with it, project two or more arms HH, which


A
B
R
Q
9.
8.
7.



support the guide-bars GG, between which slides the cross-head, not shown, to which the outer end of the piston-rod is fastened, as may be understood from the equivalent parts of nearly any locomotive or stationary engine.

Construction.—Only the principal measurements being given, the others can be assumed, or made out by the given scale. The left side of the bed having vertical faces, these may be used as lines of reference from which to lay off horizontal measurements. Vertical ones can be laid off from the base line AB; or, on the guide attachments, from the top of the guides downward. To give greater stiffness to the arm H, its lower principal curve is struck from a centre, *b*, $1\frac{1}{2}''$ to the right of *a*, the centre of its semicircular outlines. The curved outlines generally are composed of circular arcs tangent to each other.

As a minute following of given copies is not intended, these general explanations, measurements, and scale will sufficiently guide the learner in the construction of examples like the present.

Execution.—The thin material of the bed gives occasion for section lines as fine and close together as can well be made.

Exercises.—1. Reverse the figure right for left.

2. Supposing the guides to be four feet long, make a side and a plan view, showing three arms to the supporter H.

7. *Bearings.*—This is a general term meaning any surface which immediately supports a moving piece. The bearings of a rotating piece are cylindrical and variously termed. *Journals* are formed in the frame of a machine and lined with brass or other anti-friction alloy. When detached, they are *pillow-blocks*, as already shown. *Bushes* are whole hollow cylindrical linings of journals, but being unadjustable to compensate for wear, separate brasses are better. *Foot-steps* are the bearings at the base of vertical shafts, the lower end of which is a *pivot*. *Axle boxes* are the terminal supports of rail-car axles, and have a small vertical range of motion between the jaws of a stout iron frame.

Cranks and Eccentrics.

8. A crank, Fig. *a*, is an arm, CC′, keyed at one end firmly to a revolving shaft SS′ by a key *kk′*, and hence revolving

with it; and at the other end carrying a *crank-pin pp′*, which is embraced loosely by a *connecting rod*. This connecting rod similarly embraces a parallel pin in the cross-head attached to a piston-rod, a pump-rod, or other piece having a reciprocating motion. Thus a rectilinear reciprocating motion, as of a piston, is converted into a rotary motion, as seen in any locomotive, or

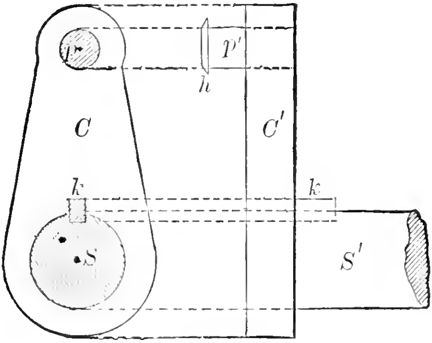


Fig. a.

a stationary engine of the usual type, or *vice versa*, as in case of a pump. The length of the *stroke* of the piston must evidently be equal to the diameter of the circle described by the centre of the crank-pin; that is, equal to twice Sp. See Fig. *b*, where the stroke qq_1 of the forward end of the connecting rod mr is equal to pp_1, the dotted circle being that described by the centre of the crank-pin.

9. Fig. *b* illustrates an important elementary point in crank motions. Remembering that any connecting rod is of invariable

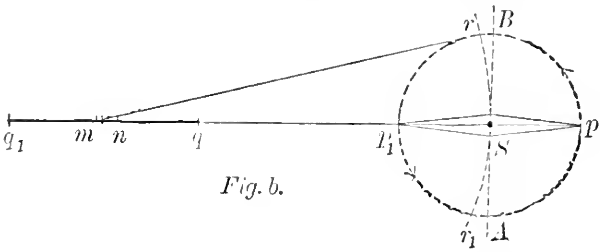


Fig. b.

length, take the middle-point m of the stroke qq_1 as a centre, and the length $m\text{S} = qp = q_1p_1$ of the rod as a radius, and the arc

rSr_1 thus described will intersect the crank-pin circle in the corresponding positions r and r_1 of the crank-pin.

Thus, while the cross-head pin passes over mq and qm, the crank-pin describes the arc r_1pr, *greater* than a semicircle; but while the former is passing over mq_1 and q_1m, the crank-pin proceeds over rp_1r_1, *less* than a semicircle.

Conversely, while the crank-pin traverses the rear semicircle ApB, the cross-head pin only travels from n to q and back; but when the crank-pin describes the semicircle Bp_1A, the other pin travels from n to q_1 and back; An being equal to r_1m.

With the use of the connecting rod, this inequality mn, between the two partial double strokes Qnq and Qnq_1, would disappear only by using a rod of infinite length. But the employment of a yoke with a slot, equal and parallel to AB, as in

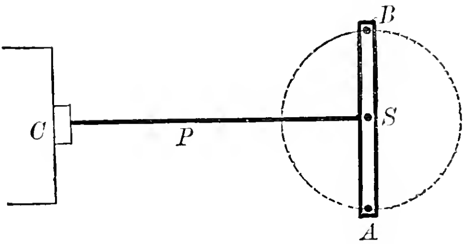


Fig. c.

Fig. c, produces the same result by finite means. Here a piston-rod, P, issuing from the steam-cylinder C, is rigidly attached to a yoke, AB, in which the crank-pin plays as it is driven by the yoke. In this case the piston is exactly at the middle point of its stroke when the crank-pin is at either end of the diameter AB. This movement is often seen in steam fire-engines.

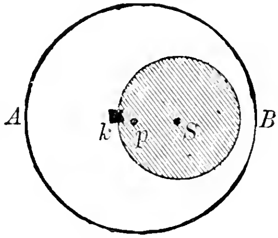


Fig. d.

10. *Eccentrics.*—The distance, Fig. a, from the centre of the shaft S to the centre of the crank-pin p, is called the *arm* of the crank. When this arm is so short, as compared with the diameter of the shaft, as to be entirely within the shaft, as at Sp, Fig. d, the crank-pin AB, whose centre is p, has to be

made large enough to embrace the shaft. In this case the crank-pin is called an *eccentric.*

That the eccentric is simply a short crank in principle and action, will be evident by substituting for the crank C, Fig. *a*, a circular plate with centre *p* and radius sufficient to include the shaft. In either form of Fig. *a*, and in Fig. *d*, a connecting rod attached to the crank-pin would actuate any piece at its opposite end through a stroke equal to twice S*p*.

11. A *connecting rod* is attached to a crank-pin by a method having many modifications in minor details. The general principle, alike for all, is shown in Fig. *e*. The object to be secured is an invariable distance between the centres of the crank-pin and the pin *p*, at which the rod R is attached to the cross-head. E is the end of this rod, called the *stub-end*. *ssss* is the *strap*,

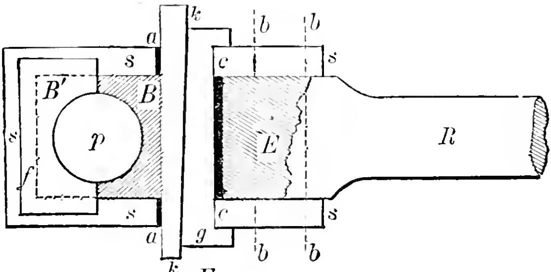


Fig. e.

in one piece. B, shown sectionally, and B′ in elevation, are the brasses, square outside and cylindrical inside, which, together, embrace the shank of the crank-pin, and are kept from sliding off by the head of the crank-pin *h*, Fig. *a*. The whole is fastened by two bolts *bb*, *bb*.

This arrangement being understood, suppose that by long wear the brasses play loosely upon the pin *p*. By driving in the slightly tapering key *kk*, its side *aa* presses the brass B against the pin *p*, the width of the slots *ac ac* in the strap permitting this to be done. Then loosening the bolts *b*, the holes for which are oblong from right to left, as seen in a plan view, a further driving in of the key *kk* operates through the hooked piece *g*, called a *gib*, to draw the strap *s* to the right, and thus draw up the brass B′ against the pin *p*.

Having understood one construction, the learner will be able

to understand all the modifications which he may notice on locomotive or other engines, such as the omission of the gib, which is unnecessary, with the bolts; a separate key for each brass; the stub-end extending to the left of *kk*, so as to wholly enclose it, when no bolts would be necessary; a screw motion at the small end of *k* for drawing, instead of hammering in the key; etc.

Ex. 4. **A Crank.** Pl. XVI., Fig. 3. This consists essentially of two collars connected by a tapering arm, the whole in one cast-iron piece. O is the centre of the 8-inch shaft, and shaft collar of diameter *ad*, 19″. P is the centre of the crank-pin, of 4″, and of its collar, of 9″ diameter. The arm CC′ is chambered as indicated by the dotted line around C′. The linear arm OP is 24″.

The surfaces of the arm flow into those of the collars as indicated in line drawings by the curved ends of the upper edge of C′, lines whose geometrical construction is unnecessary in practice, but may be found as follows, Fig. *f*. In this figure the arc $c'n'$ is that at cn, Plate XVI., Fig. 3, enlarged, and the line On corresponds to PO. Then project points of $c'n'$ upon cn, as r' at r, and rr_1 and $r'r''$ are the two projections of the horizontal circle through rr', which cuts the edge n_1o of the crank at r_1, which, projected upon the horizontal line $r'r''$, gives r''. Similarly, other points of the required curve $n'n''\ r''o'$ are found. Such curves are, however, after the full-sized construction of a few cases, to apprehend their general form, sketched by hand, as they are not essential to a working drawing.

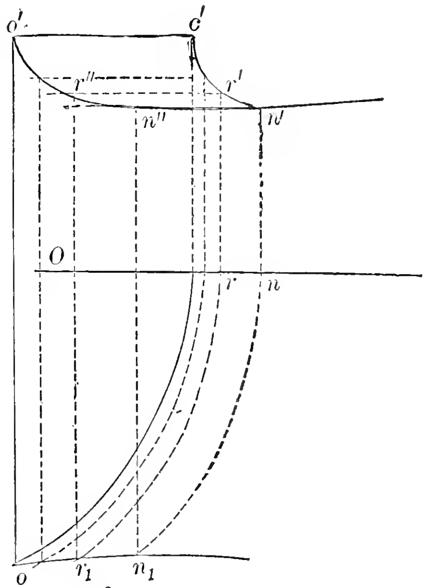


Fig. f

Exercises—1. Complete the crank, half of which is shown in the figure.

2. Make a longitudinal section of the crank.

3. Draw from measurement any accessible ribbed, trussed, or chambered crank.

4. Draw a cranked axle (such as may be seen on old locomotives having "inside connections").

Ex. 5. **A Ribbed Eccentric and Strap.** Pl. XVI., Figs. 4, 5. This may also be called an open or skeleton eccentric. O is the centre, and O*b* the radius of the shaft, $8\frac{1}{2}''$ diameter, to which the eccentric is clamped by clamp-screws, one of which is *n*. The centre of the eccentric is *a*, which makes the crank-arm O*a* of the eccentric $3''$, and hence the stroke, called the *throw*, of the valve, or whatever piece is moved by the eccentric, $6''$.

The width of the different parts of the eccentric is shown on the fragment of sectional view, as at $o'o''$ the thickness of the flange, or feather, *o*, the width $c'e''$ of the collar *b* and rim *e*, and the width of the rib *c*.

Fig. 5 shows a little more than one-quarter of the strap which surrounds the eccentric, and a little more than half of one of its two halves, which are bolted together through the ears, as *cd*. The arc *ab* is of the radius *ae*, Fig. 4; *mn* is of the radius *ac*, thus showing the groove in the strap which just fits the rib on the eccentric, and so prevents the strap from slipping off. The portion of the strap shown carries the socket *ne*, in which is keyed or clamped the eccentric rod, corresponding to the connecting rod of a crank. The opposite or left-hand half is uninterrupted in outline. The figure $r'g'$ shows the form of the section at *rg*.

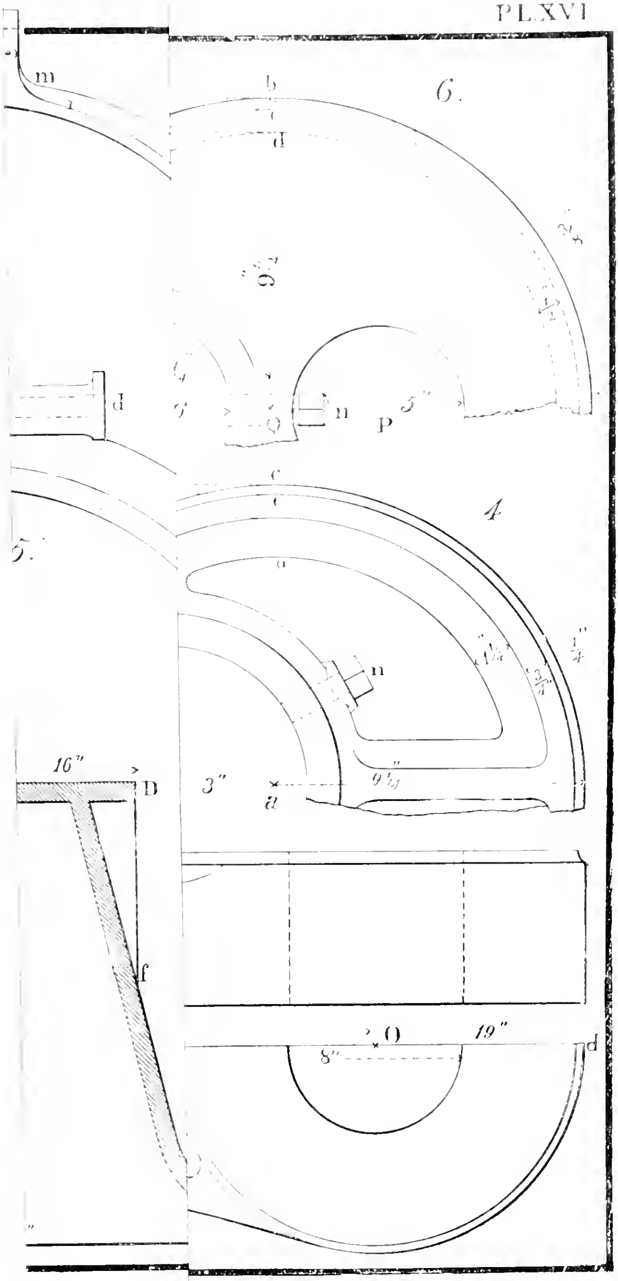
Execution.—The numerous tangent arcs and curved heavy lines tapering at their termination will afford occasion for special care.

Exercises.—1. Draw the whole of the eccentric and its strap.

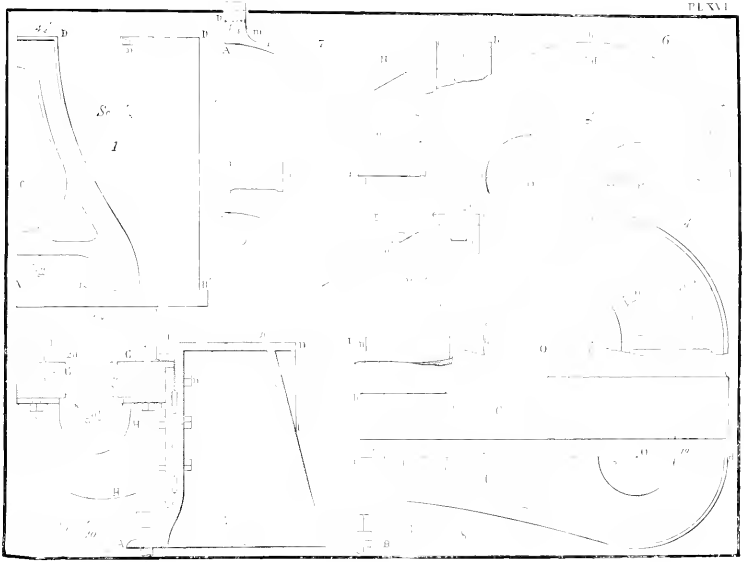
2. Make a horizontal section of the eccentric.

3. Make an end elevation of the eccentric and strap.

Ex. 6. **A Grooved Eccentric.** Pl. XVI., Figs. 6, 7. This might also be called, by reason of its form, a chambered or box eccentric, since all of it between the solid collar Q*a* and the



m
b
6.
d
9½
½
d
n
O
P
5"
c
4
5.
o
n
1¼"
¾"
¼"
16"
D
3"
a
9⅛"
f
O
19"
8"
d



rim *cd* consists essentially of two thin plates enclosing a hollow interior of width, $3\frac{5}{8}''$, shown on the fragment of end elevation —the scale is $\frac{3}{16}$, as in Ex. 5.

The shaft opening, of centre O, is $6''$ in diameter, surrounded by solid metal $1''$ thick as indicated. The *arm* OQ being $4\frac{1}{4}''$, makes the *throw* of this eccentric $8\frac{1}{2}''$. The opening, P, $5''$ diameter in the walls of the eccentric, gives access to the clamp-screw *n* by which it is fastened to the shaft.

The strap, Fig. 7, a section of which is shown at H, sets in the groove $c'c''$ of the circumference of the eccentric. Its outer arc *mn* is drawn from a centre a little to the right of that of AB, so as to support the rod-socket BC. As in the last example, the strap is in two halves bolted together through ears as at D.

Exercises.—1. Draw the whole eccentric with the strap in place upon it, and an end view of both.

2. Make a horizontal and a vertical section (perpendicular to the paper) of the combined eccentric and strap.

12. *Check, lock, or jamb nuts.*—On parts of machinery which are exposed to a jarring motion at high speed, as in locomotive machinery, two nuts are commonly seen at the end of the bolts which secure such pieces. These serve to clamp each other against the screw-threads of the bolt, and thus hold each other from working off the bolt.

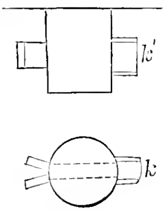


Fig. g.

Other contrivances for securing the same result, are nuts with notched sides, into which a detent enters, as may be observed in winding up a watch; or a forked key kk' through the bolt and outside of the nut, as in Fig. *g*.

CHAPTER II.

GEARING.

13. *Gearing* is the term applied to wheels or straight bars when they are armed with interlocking teeth enabling them to take a firmer hold of each other, for the purpose of communicating motion, than they could if they were smooth surfaces, tangent to each other and communicating motion only by means of the friction of their surfaces of contact.

In order to a smooth and uniform motion, the teeth must be equal and equidistant, and those on each body adapted to the form of those on the other body. Also, in order that the toothed bodies, two cylinders tangent to each other, for example, should preserve the distance between their centres, depressions *below* the original surface of each body must be made between the teeth, in order to receive the portions which project *beyond* that of the other body.

A toothed bar is called a *rack;* a toothed cylinder, a *spur-wheel,* or *pinion* if small; a toothed cone, a conical or *bevel wheel.*

14. *Forms of teeth.*—1°. When a circle C, Pl. XVII., Fig. 1, rolls, without slipping, on a fixed straight line AB, any one point of the circle describes the curve called a *cycloid.* Thus the point 0 describes the cycloid, one-half of which is 0E′, while the circle C rolls to the position C′.

2°. When, on the contrary, a straight line as 3,3′, Fig. 2, rolls, without slipping, on a fixed circle, any point of the rolling line describes the curve called an *involute* of the circle. Thus 3′ describes the involute 3′, 2′, 0.

3°. Again: when one circle rolls on the exterior of another as the circle BB′, Fig. 3, on the circumference BFA, any point on the rolling circumference traces the curve called an *epicycloid.* Thus the point F traces the half epicycloid FG, while the circle C′ rolls to the position AG.

4°. Finally: when BB′, instead of rolling on the convex or exterior side of the circumference C, rolls upon its concave side,

or within it, any point of the rolling circle generates the curve called a *hypocycloid.*

In all these cases the fixed line is called the *base line* or *circle.*

15. Suppose now the circle BB′, Fig. 3, to be revolved 180° about a tangent at B. It would then be tangent to the circle C *interiorly* at B, as it now is exteriorly at that point.

If then, the diameter BB′ being *less* than radius CB, the circle BB′ rolls within C, on the arc BFA, the hypocycloid traced by B will be *above* AB. But if a circle of diameter *greater* than CB were to roll within C on the arc BFA, the hypocycloid curve would be *below* AB. It plainly follows that the hypocycloid traced by the point B, when the circle of diameter just equal to CB rolls within C on the arc BFA, would coincide with BC. That is: *the hypocycloid traced by any point of a circumference which rolls on the inside of a base circle of twice its diameter, is a straight line.*

16. These four curves (14) are suitable forms for the teeth of wheels, for the simple reason that when two circles as C and C′, Pl. XVII., Fig. 4, maintain rolling contact with each other, as at H, equal arcs of each come in contact in a given time. Hence the motion is the same in effect as if, separately, C rolled on C′ as a fixed circle, and then C′ rolled over an equal arc on C as a fixed circle, and therefore contact will be maintained by arming the wheels with teeth generated by the point H, for each case respectively.

Similarly for a wheel C, and rack I′J′.

The circle or line employed for generating the tooth curves is called their *generating circle*, or *line.*

17. *Construction of tooth curves.*—This is very simple, and follows directly from the definitions in (14). *Cycloid.* Thus Pl. XVII., Fig. 1, the points 1, 2, 3, etc., on the circle C indicate the heights of 0 above AB, corresponding to 1, 2, 3, etc., on AB as successive points of contact of C with AB. Then the intersections of the parallels to AB through 1, 2, 3, etc., on C, with the arcs of radii *a*1, *b*2, *c*3, etc., will be points of the cycloid 0E′. Both sets of spaces 01, 12, etc., are equal, since there is no slipping of the circle in rolling on AB. *Epicycloid.* Likewise in Fig. 3, arcs F1, etc., on circle C′, = arc F1, etc., on circle C, express

the character of the motion; *a*, *b*, *c*, etc., are positions of the centre C′ corresponding to 5, 4, 3, etc., as points of contact of the circles; the arcs of radii C1, C2, C3, show the radial distances of F from BFA as C′ rolls on C; hence, finally, the intersections, not lettered, of the arcs of radii *e*1 and C1, *d*2 and C2, *c*3 and C3 are points of the epicycloids FG and FB′.

The hypocycloid is constructed in a precisely similar manner.

The *involute* is approximately represented by tangent circular arcs as in Fig. 2. Here, 11′, 22′, 33′, being positions of the rolling straight line at equidistant points of contact 1, 2, 3, we describe the arc 01′ with radius 10 (taking the chord as approximately equal to the arc), then an arc 1′ 2′ with radius 21′, then the arc 2′ 3′ with radius 32′, etc. The more numerous the points 1, 2, 3, etc., the closer will the compound curve thus found approximate to a true involute.

These curves can be constructed mechanically on a large scale by means of a pin or pencil point inserted firmly in the edge of a wooden ruler or circle, either of which is made to roll without slipping on the other, or the circle on a fixed circle; or within a circular opening in a thin board, in the case of the hypocycloid.

18. *Definitions.*—Let the circles of radii CH and C′H, Pl. XVII., Fig. 4, represent the original circumferences of two cylinders having IJ for a common tangent at H, and now provided with interlocking teeth as shown, forming a pair of *spur-wheels*. These circles are called *pitch-circles*. The corresponding line I′J′ of the *rack* is called its *pitch-line*.

The distance *ab*, or H′K on the rack, which includes a tooth and a space on the pitch-circle, is called the *pitch*.

The circle of radius C*c* is the *root-circle*, and contains the *roots* of the teeth.

The circle of radius C*d* is the *point-circle*, and contains the *points* of the teeth.

The surfaces as *bd* are the *faces* of the teeth, and those as *bc* are their *flanks*.

19. *Usual proportions.*—Supposing the pitch divided into 15 equal parts, 7 of these are taken for the width, *ah*, of the tooth, leaving 8 of them for the width of the space, *hb*, to allow easy working of the teeth. Also $5\frac{1}{2}$ of these spaces are taken for the

radial extent of the teeth beyond the pitch-circle and $6\frac{1}{2}$ of them for their depth below the pitch-circle, to prevent the tooth points of one wheel from striking the rim of the other wheel.

Application of Tooth Curves.

20. *Designing of gearing.*—Comparing (15) and (16), the generating circle of the tooth curves must be smaller than the pitch-circle in order to form the necessary flank surfaces (18). A common practice is, to employ for the flanks of each wheel a generating circle of diameter equal to the radius of the pitch-circle of that wheel; in order to produce radial flanks, as most simple. Now, as seen by inspection of Pl. XVII., Fig. 4, the *face* of a tooth of each wheel is in contact with the *flank* of some tooth of the other wheel. Hence (16) the same circle that generates the *flanks* of one wheel must generate the *faces* of the teeth of the other, since keeping the generating circle of diameter CH in contact with the pitch-circles at their point of contact H, requires in effect the equal rolling of that generating circle upon the exterior of the circumference of wheel C′, and on the interior side of that wheel C. This can easily be seen experimentally by using three card-board circles.

21. *Detailed description.—Epicycloided teeth.*—The circle CH generates the radial *flanks*, as *bc*, of the teeth of C (14, 4°), and by rolling on the exterior of C′ generates the face curves of the teeth of C′. To avoid confusion, HK may represent one of these curves, though it is really an involute. Likewise, the circle C′H generates the radial flanks of wheel C′, and by rolling on the exterior of C will generate the epicycloidal faces of its teeth, found as in Fig. 3, but represented as before by the involute HL.

22. *Objections.*—Each wheel having a separate generating circle, each will work correctly only with the other. But if one uniform generating circle be employed for the faces and flanks of any number of different-sized wheels of the same pitch, any two of them will work together properly. This common generator must not exceed half the size of the least wheel of the set, so as to avoid convex flanks (15).

23. *Involute teeth.*—Involute faces for both wheels can be

formed as shown by the rolling of the common tangent IJ at H, first on C, giving the involute face curve HL (Fig. 4), and then on C′, giving the face curve HK.

Usually, however, when involute teeth are employed, they are not combined with radial flanks, since this violates the principle that the same generatrix should form the face and the flank which are to be in contact; but IJ is made a common tangent through H to the root-circles of the two wheels, so that the involute teeth will be bounded by a single involute curve reaching to the root-circles, as they should, since a straight line cannot be rolled on the interior side of the pitch-circles to produce separate flank curves.

24. *The rack-generating circle* continuing to be half the size of its own pitch-circle, the generating circle for the rack flanks will be I′J′, since a straight line is a circle of infinite radius and half of that radius is infinite still. This understood, the *flanks* of the rack are straight lines as AH′ perpendicular to its pitch-line, and the *faces* of the teeth of C, being properly generated by the same line, are involutes as H′F′.

Likewise the *flanks* of the teeth of C are straight lines generated by the circle of diameter CH′, while the *faces* of the rack teeth are cycloids as H′G′ generated, as shown, by the rolling of the same circle on the pitch-line I′J′. But, as before, one generating circle can be used for faces and flanks of both wheels.

Ex. 7. **The Drawing of a Spur-wheel.**—It is *convenient* that the pitch should be some simple measure as 1″, 1⅛″, 1¼″, . . . 2″, . . . 2½″, . . . etc., and it is *necessary* that the pitch should be contained an exact number of times in the pitch-circle. Hence the usual problem is: Given the pitch and number of teeth of a pair of wheels, to find their radii.

Let P = pitch N, = number of teeth, R = radius,

and C = circumference of pitch-circle.

Then $C = P \times N = 3.1416 \times 2R$,

whence $R = \dfrac{P \times N}{2 \times 3.1416}$, or denoting as usual, 3.1416 by π

$$R = \frac{P \times N}{2\,\pi}.$$

Suppose a wheel of 24 teeth and $1\frac{1}{2}''$ pitch. Its circumference will thus be $36''$ and its radius very nearly $5\frac{3}{4}''$.

The four quarters of the wheel being alike, it is sufficient to draw one of them, with the first tooth on the adjacent quarters, and this can conveniently be done on a scale of half the full size.

Three forms of wheels are in use according to their size: *solid* wheels, as in Pl. XVII., Fig. 4; *plate* wheels, consisting of a central *hub*, or *boss*, keyed to the shaft, and connected by a thin *plate* to the *rim* which carries the teeth; and *armed* wheels, in which the *boss* is connected with the *rim* by arms the perpendicular section of which is often an equally four-armed cross.

Attending at first principally to the teeth, let the wheel now drawn be solid.

Divide a quadrant of the pitch-circle carefully into *six* equal parts, one of which will be the pitch.

Proportion the teeth by (19), giving the root and point circles. Lay off *half* the width of a tooth on each side of each point of division of the pitch-circle, which will make the lines as CH′ and CD, Fig. 4, centre lines of teeth instead of as shown in that figure.

While teeth are shown in detailed working drawings, of full size and by the most accurate construction of their proper forms, they are approximately represented in general illustrative drawings, by various simple methods. Thus the faces may well be drawn by taking the pitch *ab* as a radius, with the centre, as at *b*, on the pitch-circle to draw the face *a*D. A more summary process is shown in Pl. XIX., Fig. 6. The flanks, if not radial, as shown in the figure, should be in reality hypocycloids (14, 15) which would *diverge towards the centre* C, and which may sufficiently be represented by taking *d*, for example, for the centre of the flank beginning at *n*.

Thus the *elevation* may be completed, placing the shade, or heavy lines, on each tooth by the usual rule, as shown.

For the *plan*, draw two parallel lines at a distance apart equal to the *width* of the wheel; that is, the length of the teeth, which may be twice the pitch. Then simply project down the point angles as *d*, and visible root angles as *c*, and the points of contact of the face curves with tangents parallel to IJ, as at *a* and

near h. To become familiar with the subject, work out fully the following:

Exercises.—1. Construct the half, not shown, of the cycloid. Pl. XVII., Fig. 1.

2. Complete both of the epicycloids half-shown in Fig. 3, one with the diameter of C′ equal to the radius of C. Also one, given by making the circles C and C′ equal.

3. Construct the hypocycloid generated by the point H of circle C′H, Fig. 3, in rolling within circle C.

4. Construct the hypocycloid generated by the circle CH′ rolling within the small circle C′.

5. Construct an arc of the involute of circle C, generated by the point H′ of the line I′J′, and by dividing a quadrant of C into eight equal parts.

6. Draw a spur-wheel and rack, the wheel having 32 teeth and 2″ pitch. Make the drawing of full size, showing a quadrant only of the wheel, bisected at its point of contact with the rack, and let the faces and flanks of both pieces have one generating circle whose diameter shall be $\frac{3}{4}$ that of the radius of the wheel.

7. In Ex. 6, substitute for the rack a wheel of 20 teeth, and let the common generating circle of the teeth-profiles of both wheels be of less *diameter* than the *radius* of the smaller wheel.

8. Draw enough of a four-armed wheel of 30 teeth and $1\frac{1}{4}$″ pitch to show two arms fully, making the thickness of the *rim*, and of the *arms*, and of the *feather*, and their width also (see o, Pl. XVI., Fig. 4), which surrounds the openings between the arms, all equal $\frac{7}{16}$ of the pitch, and the radial thickness of the hub $\frac{7}{8}$ of the pitch.

9. Draw Pl. XIX., Fig. 6, twice its present size or larger, and first with involute teeth, and then with epicycloidal faces and hypocycloidal flanks, and after constructing carefully one tooth-profile, find by trial the centre and radius of the circular arc which will most nearly coincide with it, to use in drawing the other teeth.

25. *Velocities.*—It is clear (13) that if one wheel has 30 teeth and another 60, the former must make *two* revolutions to one of the latter, also that the radius of the former is *one half* that of the latter. What is true for one such case is evidently true in principle for all cases. That is, the number of revolutions in a given time of each of a pair of toothed wheels is inversely as its number of teeth, or as its radius.

The *circumference velocities*, as at the point of contact H, Pl. XVII., Fig. 4, are necessarily equal, but the velocities at the

same distance from the centre, as 1 foot, on both wheels are as the numbers of revolutions, and hence inversely as their radii. The latter are termed *angular velocities.* Then denoting them by V and v for the wheels C and C′ respectively, and the radii CH by R and C′H by r, we have

$$V : v :: r : R.$$

Bevel and Mitre Wheels.

26. Pl. XVIII., Fig. 1, shows a pair of *bevel wheels.* These consist of a pair of frusta of cones, CAD and one of which CAB is the half, provided with teeth which converge to the common vertex, C, of the cones, whose axes, CB and CF, may make any angle with each other.

When, as in Fig. 2, the axes are at right angles, the wheels are distinguished as *mitre wheels.*

As C is lowered nearer and nearer to AB, still continuing the common vertex of the cones, the wheel AB becomes flatter and flatter, and when finally C passes below AB, the wheel AB becomes a hollow frustum toothed on its *inner surface.*

On account of the intersection of the axes of bevel wheels, one or both of the axes terminate at the wheels, as in Figs. 1 and 2.

27. *Velocities.*—The principles of (25) apply to bevel wheels. Hence having given one wheel, as CAB, Fig. 1, and the ratio of the velocities, make $a'd'$ and $a'e'$ in this ratio, $a'd'$ representing the relative velocity of the required wheel, and Ce' will be the axis tangent from C to an arc of centre a' and radius $a'e'$, and AD the diameter of the latter wheel.

Or, having given the vertex C, and axes CB and CF, set off Ce' and Cb' inversely as the two velocities (that is, set off on each axis a distance proportional to the velocity of the other axis), and complete the parallelogram $Cb'a'e'$, and CA is the line which will divide the angle BCF included by the axes, so as to give the radii AB and $\frac{1}{2}$AD of the required wheels.

When, as in Fig. 2, the axes are at right angles, the latter construction applies, but the parallelogram becomes the rectangle $Cba0$.

EX. 8. **To Draw a Pair of Bevel Wheels.** Pl. XVIII., Figs. 2–5.

Let the cones CAB and CAD—called the pitch-cones, because they contain the pitch-circles—be given. At A, the point of contact of the pitch-circles, draw EAF perpendicular to CA, and draw EA, EB, FA, FD. Then EAB and FAD will be the cones containing the larger, or outer ends of the teeth. Next, laying off AI equal to the length of a tooth, and drawing IR parallel to AD, III parallel to AB, and GIJ parallel to EF, we have JIR and GIH. the cones containing the inner ends of the teeth.

The wheels here shown have respectively 36 and 28 teeth. Then divide each quadrant of the semi-pitch-circle on A′B′ into 9 nine equal parts, and each quadrant of the semi-pitch-circle on A″D′ into 7 equal parts. Taking the proportions before used, make B*e* and B*b*, each on BE, respectively 5½ and 6½ *fifteenths* of the pitch, to obtain the point and root circles parallel to AB through *e* and *b*, since the real height *be* of the teeth is shown in its real size on the extreme element EB of the cone EAB. The horizontal projections of these circles are those with radii C′*e*′ and C′*b*′.

The corresponding inner point and root circles are found by noting *g*, the intersection of *e*C and GH, and that of *b*C with GH. This last point is horizontally projected at *n*′.

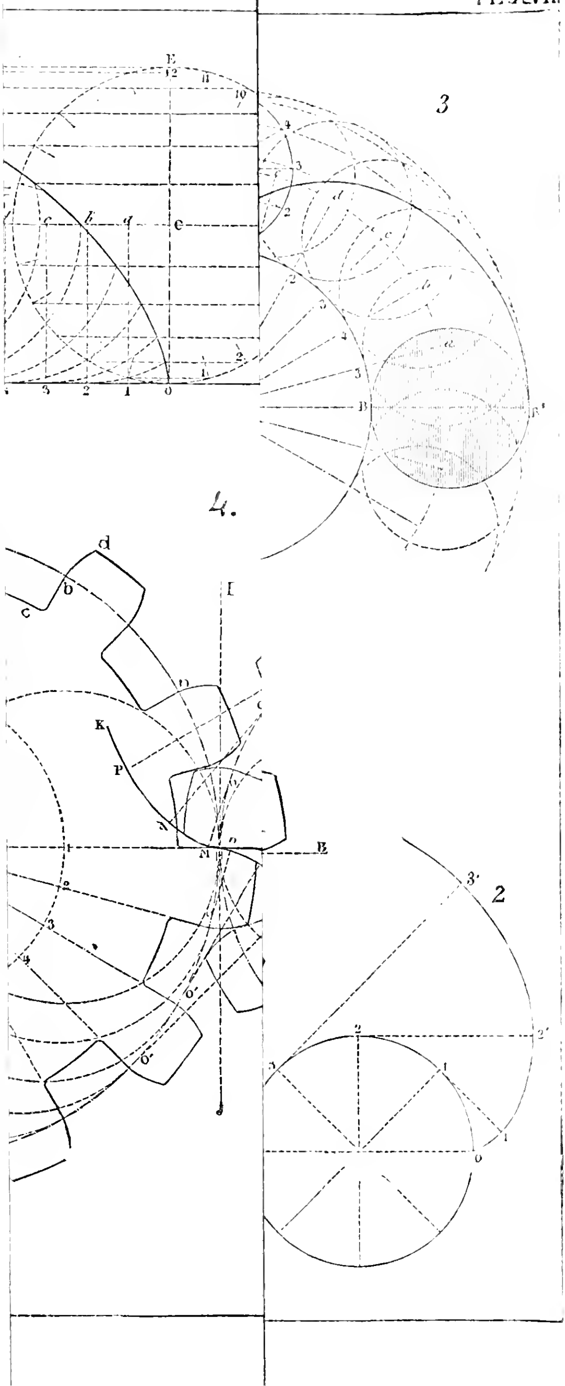
Having thus both projections of all the circles of construction:

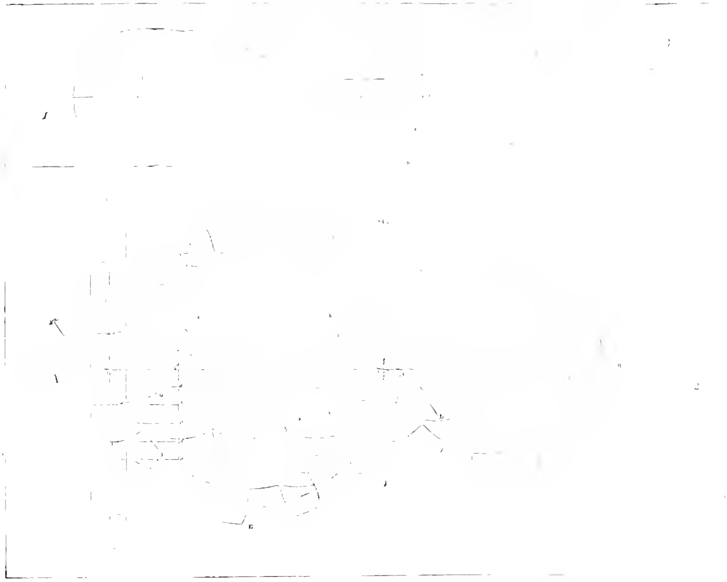
1°. Lay off $\frac{4}{15}$ of the pitch, that is, half the space between two teeth, on each side of A′, B′ and S, and from the points so found lay off the pitch, over and over, which will give all those points of the teeth which are in the outer pitch-circle A′SB′; and project these points on AB.

2°. Through the points just found on A′SB′ draw lines to C′, limited by the circle C′*n*′; and through those on AB, lines to E limited by *ab*, for the outer ends of the flanks.

3°. From the points on *ab* draw lines to C, limited by the vertical projection of circle C′*n*′, for the root lines of the teeth, and from the points thus found, the inner ends of the flanks radiating from G and limited by HI.

4°. Make arcs, tangent to each other as at O, Fig. 5, with radii EA and FA, which will be (Div. I., Prob. 28) arcs of


3
4.
2



the developments of the outer pitch-circles of the two wheels. On these lay off the pitch, and proportion the teeth as already described, the flanks running to E (below the border) and F, and the faces drawn with convenient circular arcs to replace the epicycloidal curves OP and OQ. Having thus found the width of the teeth at their outer points, lay off half this width on each side of the middle point of each tooth on the circle of radius $C'e'$.

5°. Through these points on circle $C'e'$ draw lines to C', limited by circle $C'g'$; project the points of circle $C'e'$ upon ec, vertical projection of circle $C'e'$, and thence draw the point edges of the teeth towards C.

6°. Finally, the face curves at both ends of the teeth are sketched by hand, tangent to the flanks.

By precisely similar operations, the two projections of the wheel AD—A″D′ may be drawn.

The hub and arms of both can be easily drawn, as shown.

To become perfectly familiar with the operations here described, work out the following variations :

Exercises.—1. Changing the numbers of teeth, let the axis of the wheel AD be perpendicular to the paper at C, so as to appear as Fig. 4 now does.

2. Again changing the number of teeth, let the wheel AD be in gear with AB at BH, and then draw the figure as if Pl. XVIII. were upside down, making C′A′SB′ the elevation, instead of, as now, the plan.

Screws and Serpentines.

28. *Triangular-threaded screws.*—If the isosceles triangle $cd12$, Pl. XIX., Fig. 1, whose base is in the vertical line Eh, be revolved, together with that line, uniformly, around the vertical AB as an axis, having also a uniform vertical motion on Eh, it will generate the spiral *solid* called the *thread* of a triangular-threaded, also called a V-threaded screw. The *surfaces* generated by $d12$ and $c12$ are *helicoids*, upper and lower. The *lines* generated by the points c, d and 12 are *helices*, inner and outer. Eh will generate a cylinder, called the *core* or *newel* of the screw.

29. *Square-threaded and other screws.*—If, Pl. XIX., Fig. 2,

a square, EC6, be substituted for the triangle, the result will be a square-threaded screw.

If, Fig. 3, a sphere whose centre describes a helix be the generatrix, the resulting solid will be that called a *serpentine*. This is the form of a spiral spring formed of circular wire; also of the hand-rail of circular stairs, when the rail has a circular section made by cutting it "square across."

Again: if, Fig. 5, the profile of a tooth be taken as the generatrix of the thread, there will be formed the kind of toothed wheel called an *endless screw*, since its constant rotation in one direction will actuate the wheel L. It is always the screw that is the "*driver*" and actuates the wheel, which is the "*follower*," and receives a very slow motion; since the tooth G will be carried to the position of the next tooth above it, by one complete revolution of the screw.

30. *Number of threads.*—In Pl. XIX., Fig. 1, one helical revolution of the generating triangle brings the side $c12$ to the position $d24$, which allows no intermediate position of the triangle. The screw is therefore single-threaded. The like is true of the screws in Figs. 2 and 5.

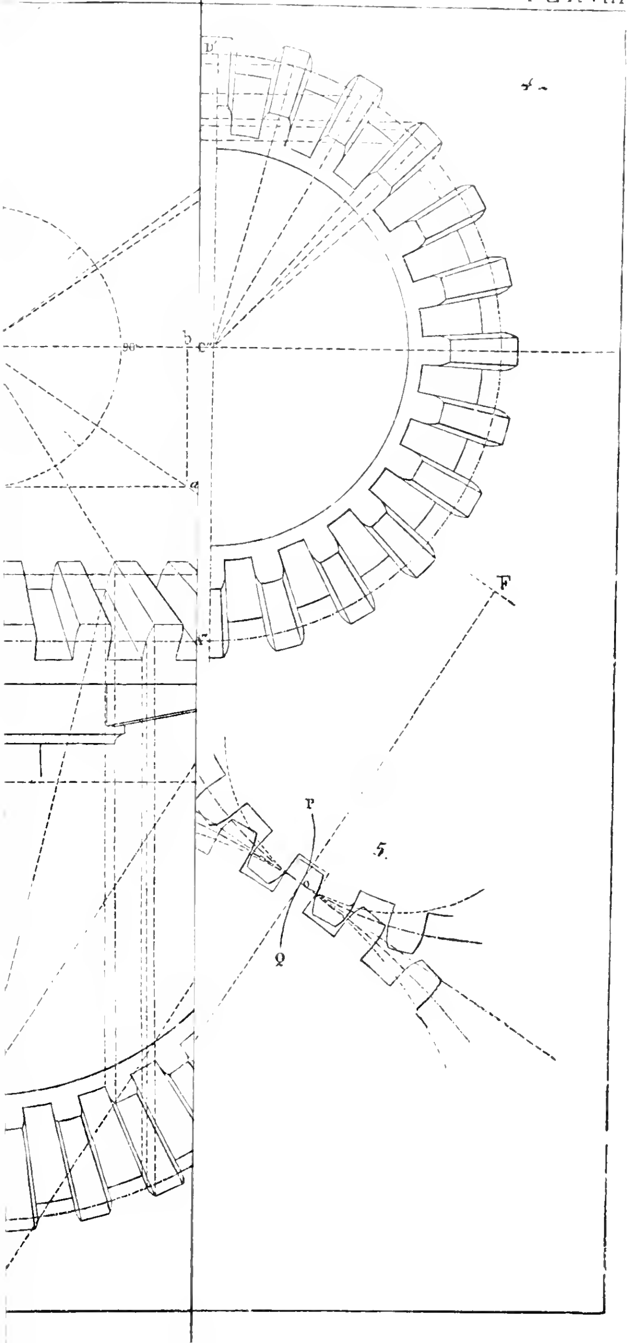
If, however, Fig. 1, one such revolution of $cd12$ had, by means of a greater ascending motion, brought $c12$ to the position rs, the screw would have been *two-threaded;* and if to the position no, it would have been *three-threaded.* The like again is true of other screws, the number of threads being adapted to the advance parallel to AB, of any point of the screw in one revolution. This advance is called the *pitch* of the screw.

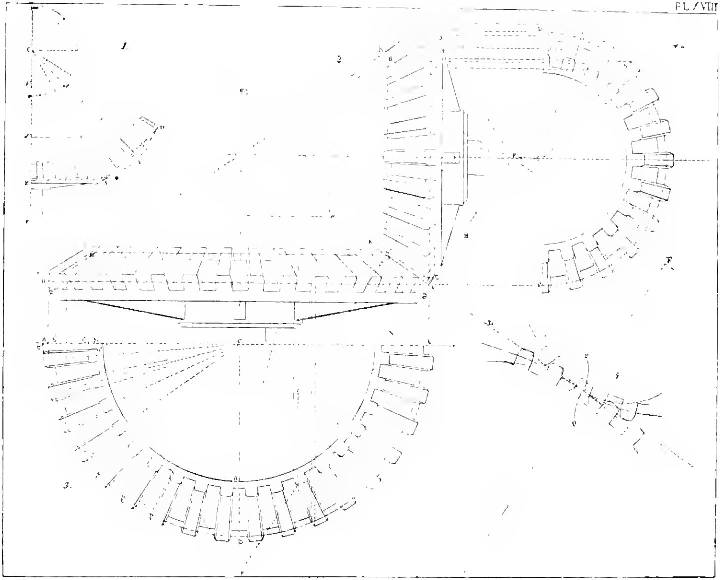
Evidently the coils of a second spiral like Fig. 3 could be laid between those shown. It would then be two-threaded.

Ex. 9. **To Construct the Projections of a Triangular-threaded Screw.** Pl. XIX., Fig. 1.

The construction of the screw consists principally in that of its helices. Accordingly, let AE and AC be the radii of the circles which represent the circular motions of the points c and 12, and which are the horizontal projections of the inner and outer helices. And let 0,12 be the pitch of the screw.

As both component motions, circular and rectilinear, of the


F
P
5.
Q



compound helical motion are *uniform*, divide the circles AE and AC and the pitch 0,12 all into the same number of *equal* parts, here 12, and draw horizontal lines through the points of division on 0,12. Then for an outer helix, project C at 0; 1 on the first horizontal above it; 2 on the second horizontal, 6, at 6 on the sixth horizontal, and so on till C is projected again at 12.

Proceed in a precisely similar manner, beginning by projecting E at *c*, to find points of an inner helix. The lines as *c*12 and *d*12 complete the figure.

Each half, to the right and left of AB, of the visible front half, as 06, of an outer helix is like the other half reversed, both right for left and upside down. Hence, as all the outer helices are alike, the portion of an irregular curve which will fit one half of one, will serve in ruling them all. Similar remarks apply to the inner helices.

Had the ascent been from D to the left on the front half of the screw instead of from C to the right, the screw would have been left-handed. Left-handed screws are only employed for special purposes, as when two rods, placed end to end, are to be separated or brought together by a screw link working on both, as seen in the truss-rods under rail-car bodies. In this case the screw-threads on one rod would be right-handed, and those on the other left-handed.

Exercises.—1. Construct the projections of a two-threaded and of a three-threaded triangular screw.

2. Construct the projections of a two-threaded and of a three-threaded left-handed screw.

Ex. 10. **To Draw a Square-threaded Screw.** Pl. XIX., Fig. 2.

The operations in this case are so similar to those of the last problem, as is evident from the figure, that they need no detailed description. The form of the thread renders the under outer helices of the left side, and the upper outer helices of the right side, of the screw visible on the back half of the screw until they disappear behind the cylindrical core. Also, the inner helices are visible only on the under left-hand side and upper right-hand side of the thread.

In the *execution*, it is very important to remember that any

one helix is, on the screw itself, of uniform curvature throughout. hence though very sharply curved in projection at the extreme points, as 6 and 12, especially in a single-threaded screw. they are not there pointed, except in drawings on a small scale where they may be approximately represented by straight lines, as in Figs. 7, 9, and 10.

Exercise.—Draw a square-threaded screw with three threads, and show all four helices of one thread throughout, but dotted where invisible.

Ex. 11. **To Draw the Interior of a Nut or Internal Screw.**

Pl. XIX., Fig. 8, shows the interior of one half of the nut for a square-threaded screw ; that is, of the hollow cylinder with a thread on its interior surface, adapted to work in the spaces between the threads of the screw. The figure representing the rear half of the nut, the threads must there ascend to the left, as they do on the rear half of the screw.

Exercises.—1. Draw the vertical section of the nut corresponding with Fig. 1.

2. Draw that of the nut of a square-threaded screw of two threads.

Ex. 12. **To Draw the Endless Screw and Worm Wheel.** Pl. XIX., Figs. 4, 5.

The profile of a tooth here becomes the generatrix of a screw-thread bounded by helices found as before. The pitch-line MN is divided by the pitch as in the case of a rack, the pitch of the screw and wheel being the same.

The wheel, having its axis in a direction perpendicular to that of the screw, is in reality a short piece of a screw having a very great pitch. That is, the angle made by the helices of the wheel-teeth with a plane perpendicular to the axis of the wheel, that is, with the plane of the paper, is the complement of the angle made by the screw helices with a plane perpendicular to *its* axis, that is, to a plane perpendicular to the paper on GD. The curves, as that to the left of N, which represent the further ends of the teeth, are assumed, unless the width of the wheel is shown by a plan view.

Ex. 13. **To Draw a Serpentine.** Pl. XIX., Fig. 3. This surface is one which, like a thin helical tube, would inclose, tangentially, all the positions of a sphere, indicated by the dotted circles, whose centre should describe a helix, ACB—2345.

The *contours,* or apparent bounding lines, of the serpentine are not helices, though at a uniform perpendicular distance from the central helix, but are drawn tangent to the numerous equal dotted circles having their centres on the helix, and which represent as many positions of the generating sphere.

Surfaces which, like the sphere and serpentine, are nowhere straight, are call *double-curved.* Where partly convex, as on the outer circle, or in the circle OF, and partly concave, as on the inner side, or on the circle OD, the contour vanishes into the surfaces, at certain points, when shown by a line drawing, as is seen at the left of the under contours, and the right of the upper ones.

The lower coil is shown *approximately* as straight, indicating what would be permissible in rough drawings or on a small scale.

DIVISION SIXTH.

SIMPLE STRUCTURES AND MACHINES.

---

237. *Note.* The objects of this DIVISION are, to acquaint the student with a few things respecting the drawing of whole structures which are not met with in the drawing of mere details; to serve as a sort of review of practice in certain processes of execution; and to afford illustrations of parts of structures whose names have yet to be defined. Proceeding with the same order as regards material that was observed in DIVISION SECOND, we have :—

CHAPTER I.

STONE STRUCTURES.

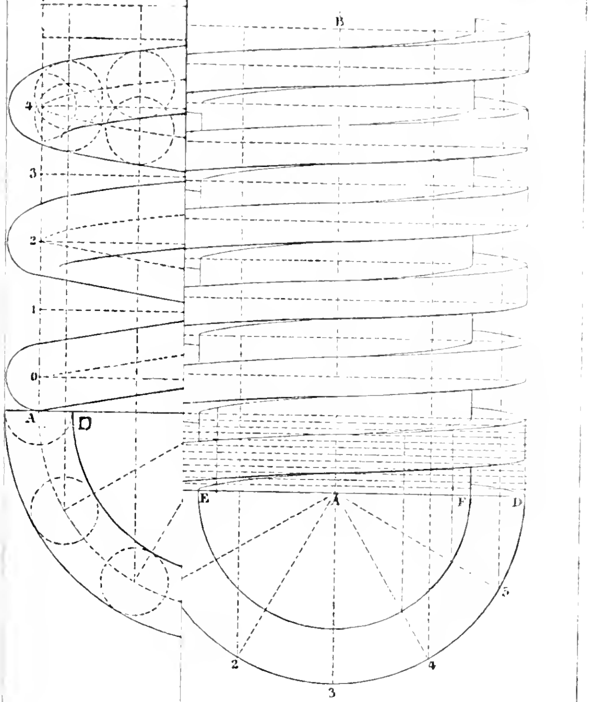
238. EXAMPLE 1°. **A brick segmental Arch.** Pl. XX., Fig. 123. *Description of the structure.*—A segmental arch is one whose curved edges, as aCc, are less than semicircles. A brick segmental arch is usually built with the widths of the bricks placed radially, since, as the bricks are rectangular, the mortar is disposed between them in a wedge form in order that each brick with the mortar attached may act as a wedge; while if the length of the bricks be radial, the mortar spaces will be inconveniently wide at their outer ends, unless the arch be a very wide one, or unless it have a very large radius.

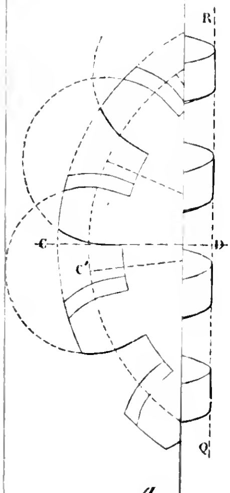
The *permanent* supports of the arch, as nPT, are called *abutments*, and the radial surface, as nab, against which the arch rests, is called a *skew-back*.

The temporary supports of an arch while it is being built are called *centres* or *centrings*, and vary from a mere curved frame made of pieces of board—as used in case of a small drain or round

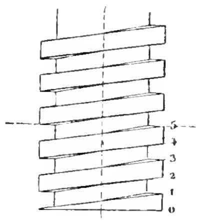
PL.XIX.

3

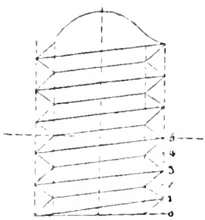




4.

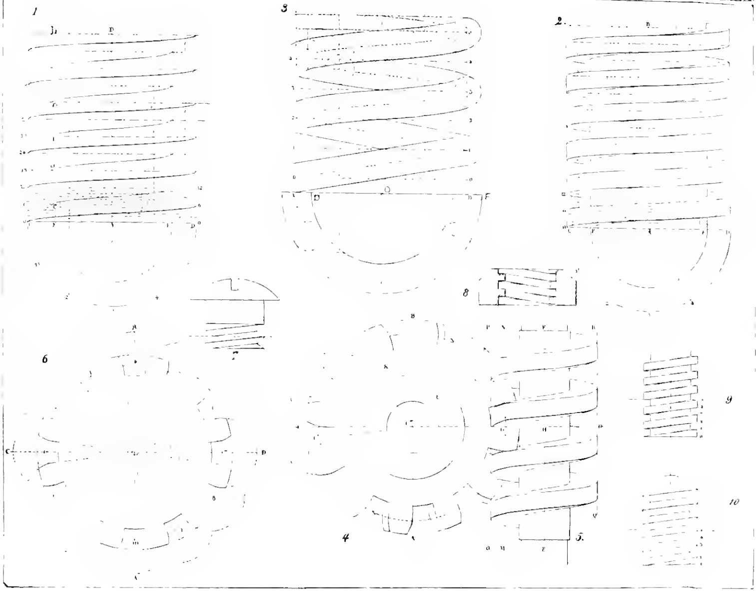


9



10.

PL.XIX.



topped window—to a heavy and complicated framing, as used for the temporary support of heavy stone bridges.

Note. The general designing of these massive centrings may call for as much of scientific engineering *knowledge*, and their details and management may call for as much practical engineering skill, as does the construction of the permanent works to which these centrings are auxiliary. In short, the *detailed* design and management of auxiliary constructions, in general, is no unimportant department of engineering study.

The span is the distance, as *ac*, between the points of support, on the under surface of the arch. The stones over the arch and abutment, form the *spandril*, or *backing*, Q*d*P.

239. *Graphical construction.*—Let the scale be one of four feet to the inch=48 inches to one inch=$\frac{1}{48}$. Draw RT to represent the horizontal surface on which the arch rests. Let the radius of the inner curve of the arch be 7 feet, the height of the line *ac* from the ground 2 feet 8 inches, and the span 7 feet. Then at some point of the ground line, draw a vertical line, OC, for a centre line; then draw the abutments at equal distances on each side of the centre line, and 6 feet 8 inches apart. Let them be 2 feet 6 inches wide.

Since the span and radius have been made equal, O*b* and O*d* may be drawn, in this example, with the 60° triangle. Drawing these lines, and making O*a*=7 feet, make *ab*=one foot, draw the two curves at the end of the arch, and make *b* and *d* points in the top surfaces of the abutments.

To locate the bricks, since the thickness of the mortar between the bricks, at the inner curve of the arch, would be very slight, lay off two inches on the arc *a*C*c* an exact number of times. The distance taken in the compasses as two inches, may be so adapted as to be contained an exact number of times in *a*C*c*, since the thickness of the mortar has been neglected, but would in practice be so adjusted, as to allow an exact number of whole bricks in each course.

The arch being a foot thick, there will be three rows of bricks seen in its front. Draw therefore two arcs, dividing *ab* and *cd* into three spaces of four inches each, and repeat the process of division on both of them.

Having all the above-named divisions complete, fasten a fine needle vertically at O, and, keeping the edge of the ruler against it, to keep that edge on the centre without difficulty, draw the lines which represent the joints in each of the three courses of brick.

240. Ex. 2°. **A semi-cylindrical Culvert, having vertical quarter-cylindrical Wing Walls, truncated obliquely.** Pl. XX., Fig. 124.

Description of the structure.—A culvert is an arched passage, often flat bottomed, constructed for the purpose of carrying water under a canal or other thoroughfare. Wing walls are curved continuations of the vertical flat wall in which the end of the arch is seen. Their use is to support the embankment through which the culvert is made to pass, and to prevent loose materials from the embankment from working their way or being washed into the culvert. Partly, perhaps, for appearance's sake, the slope of the plane which truncates the flat arch-wall, called the *spandril* wall, and the wing walls, is parallel to the slope of the embankment. The wing walls are often terminated by rectangular flat-topped posts—"piers" or "buttresses," AA', and the tops, both of these piers and of the walls, are covered with thin stones, $abcd$—$a''b''c''d''$, broader than the wall is thick, and collectively called the *coping.*

Since the parts of stone structures are not usually firmly bound or framed together, each course cannot be regarded as one solid piece, but rather each stone, in case, for instance, of the lowermost course, rests directly on the ground, independently of other stones of the same course, hence if the ground were softer in some spots, under such a course, than in others, the stone resting on that spot would settle more than others, causing, in time, a general dislocation of the structure. Hence it is important to have what are called continuous bearings, that is, virtually, a single solid piece of some material on which several stones may rest, and placed between the lowest course and the ground.

Timbers buried away from the air are nearly imperishable; hence, timbers laid upon the ground, if that be firm, and covered with a double floor of plank, form a good foundation for stone structures; and in the case of a culvert, if such a flooring is made continuous over the whole space covered by the arch, it will prevent the flowing water from washing out the earth under the sides of the arch.

When the wing walls and spandril are built in courses of uniform thickness, the arrangement of the stones forming the arch, so as to bond neatly with those of the walls, offers some difficulties, as several things are to be harmonized. Thus, the arch stones must be of equal thickness, at least all except the top one, and then, there must be but little difference between the widths of the top, or *key stone*, and the other stones; the stones must not be disproportionately thin or very wide, they should have no re-entrant

angles, or very acute angles, and there must not be any great extent of unbroken joint.

241. *Graphical construction.*—Let the scale be that of five feet to an inch=60 inches to an inch$=\frac{1}{60}$.

a. Draw a centre line, BB′, for the plan.

b. Supposing the radius of the outer surface, or back, of the arch to be $5\frac{1}{2}$ feet, draw CC′ parallel to BB′ and $5\frac{1}{2}$ feet from it.

c. Draw BE, and on C′C produced, make EC=9 feet 8 inches, CD, the thickness of the face wall of the arch=2 feet 4 inches, and the radius, *o*D, of the face of the wing wall=4 feet.

d. With *o* as a centre, draw the quadrants CG, and DF, and with a radius of 3 feet 8 inches, draw the arc *ch*, the plan of the inner edge of the coping. Also draw at D and C, lines perpendicular to BB′ to represent the face wall of the arch.

e. At G, draw G*h* towards *o*, and =3 feet, for the length of the cap stone of the buttress, AA′, and make its width =2 feet 10 inches, tangent to CG at G. The top of this cap stone, being a flat quadrangular pyramid, draw diagonals through G and *h*, to represent its slanting edges.

f. Supposing the arch to be $1\frac{1}{2}$ feet thick, make C′H=$1\frac{1}{2}$ feet, and at C′ and H, draw the irregular curved lines of the broken end of the arch, and the broken line near the centre line, also a fragment of the straight part of the coping.

g. Let the horizontal course on which the arch rests, be 2 feet 9 inches wide, i. e., make H*e*=3 inches, and C′*n*=1 foot; and let the planking project 3 inches beyond the said course, making *er*=3 feet. Through *e*, *n* and *r*, draw lines parallel to BB′ and extending a little to the right of C′H.

h. Proceeding to represent the parts of the arch substantially in the order of their distance from the eye, as seen in a plan view, a portion of the planking may next be represented. The pairs of broken edges, and the position of the joints, show that there are two layers of plank and that they break joints.

i. Under these planks, appear the foundation timbers, which being laid transversely, and being one foot wide and one foot apart, are represented by parallels one foot apart, and perpendicular to BB′. Let the planking project 4 inches beyond the left hand timber. Observe that two timbers touch each other under the arch front.

j. The general arrangement of stones in the curved courses of the wing wall, in order that they may break joints, is, to have three and four stones, respectively, in the consecutive courses. To indicate

this arrangement in the plan, hG, fb, gd and DC will represent the joints of alternate courses, and the lines km, &c. midway between the former, will represent the joints of the remaining intermediate courses.

This completes a partial and dissected plan which shows more of the construction than would a plan view of the finished culvert, and as much, as if the parts on both sides of the centre line were shown. In fact, in drawings which are strictly working drawings, each projection should show as much as possible in regard to each distinct part of the object represented.

242. *Passing to the side elevation*, which is a sectional one, showing parts in and beyond a vertical plane through the axis of the arch, we have:—

a. The foundation timbers, as $m'q$, &c., projected up from the plan; or, one of them being so projected, the others may be constructed, independently of the plan, by the given measurements.

b. The double course of planking op, appears next with an occasional vertical joint, showing where a plank ends.

c. The buttress, A, and its cap stone Y, are projected up from the plan, and made 6 feet high, from the planking to G′.

d. From G′ and h', the slanting top of the wing walls are shown, as having a slope of $1\frac{1}{2}$ to 1—i. e. $h'h''=\frac{3}{2}\,h''u$—and the vertical lines at C′, D′ and D″ are projected up from C, D and D‴.

The remaining lines of the side elevation are best projected back from the end elevation, when that shall have been drawn.

243. *In the end or front elevation*, we have:—

a. At $m''m'''$, a side view of one of the foundation timbers, broken at m''', so as to show other timbers behind it.

b. The planking $o'o''$ in this view, shows the ends of the planks in both layers—breaking joints.

c. $No'=Bo'''$, taken from the plan; and in general, all the horizontal distances on this elevation, are taken from the plan, on lines perpendicular to BB′.

d. The vertical sides of the buttress, A′, are thus found. The heights of its parts are projected over from the side elevation.

e. The thickness of the foundation course, $ts=1\frac{1}{2}$ feet, and $tr'=en$, on the plan.

f. The centre, O, of the face of the arch, is in the line $r't$ produced. The radius of the inner curve (intrados) of the arch is 4 feet and of the cylindrical back, behind the face wall, $5\frac{1}{2}$ feet—shown by a dotted arc. In representing the stones forming the arch, it is to be remembered that they must be equal, except the "key stone,"

g, which *may* be a little thicker than the others; they must also be of agreeable proportions, free from very acute angles, or from re-entrant obtuse angles; and must interfere as little as possible with the bond of the regular horizontal courses of the wing walls. There must also be an odd number of stones (ring stones) in the front of the arch.

On both elevations, draw the horizontal lines representing th wing wall courses as one foot in thickness, and divide the inner curve of the arch into 15 equal parts. Draw radial lines through the points of division. Their intersections with the horizontal lines are managed according to the principles just laid down.

g. The points, as k and f, in the plan, are then projected into the alternate courses of the side elevation, and into the line, Bo''', of the plan.

From the latter line, the several distances, $o'''b'''$, &c., from o''', thus found, are transferred to the line $o'N$, as at $o'b''''$, &c., and at these points the vertical joints of the front elevation are drawn in their proper position, as being the same actual joints, shown by the vertical lines of the side elevation. In the stones immediately under the coping, there must generally be some irregularity, in order to avoid triangular stones, or stones of inappropriate size.

h. To construct the front elevation of the coping. All points, as a, a', a'', in either the front or back, or upper or lower edges of the coping, are found in the same way, and as follows:

a'' is in a horizontal line through a' and in a line $a''a''''$, whose distance from o' equals the distance $o'''a'''$ on the plan. Constructing other points similarly, the edges of the coping may be drawn with an "irregular curve."

The horizontal portion of the coping, over the arch, is projected over from C′ and from the two ends of the vertical line at D′.

Execution.—In respect to this, the drawing explains itself.

Example. Let this design, or any similar one, be drawn on a scale of four feet to the inch, on a larger plate; not forgetting to place the three projections in their proper relative position, as shown (15) and (32).

CHAPTER II.

WOODEN STRUCTURES.

244. Ex. 3°. Elevation of a "King Post Truss."

Mechanical construction, &c.—A Truss is an assemblage of pieces so fastened together as to be virtually a single piece, and therefore exerting only a vertical force, due to its weight, upon the supporting walls.

In Pl. XX., Fig. 125, A is a *tie beam;* B is a *principal;* C is a *rafter;* D is the *king post;* E is a *strut;* F is a *wall plate;* G is a *purlin*—running parallel to the ridge of the roof, from truss to truss, and supporting the rafters. H is the *ridge pole;* W is the wall, and *ab* is a strap by which the tie beam is suspended from the king post.

245. *Graphical construction.*—In the figure, only half of the truss is shown, but the directions apply to the drawing of the whole. In these directions an accent, thus ′ , indicates feet, and two accents, ″ , inches. For practice draw the whole figure, and on a larger scale.

a. Draw the vertical centre line *b*D.

b. Draw the upper and lower edges of the tie beam, one foot apart, and 12′ in length, on each side of the vertical line.

c. On the centre line, lay off from the top of the tie beam, 5′—6″ to locate the intersection of the tops of the principals; and on the top of the tie beam, lay off 11′ on each side, to locate the intersection of the upper faces of the principals with the top of the tie beam.

d. Draw the line joining the two points just found, and on any perpendicular to it, as *fg*, lay off its depth $=8''$, and draw its lower edge parallel to the upper edge. Make the shoulder at $o=3''$ and parallel to *fg*.

e. From the top of the beam, draw short indefinite lines, *c*, 6″ each side of the centre line, and note the points, as *e*, where they would meet the upper sides of the principals.

f. Draw vertical lines on each side of the centre line and 4′ from it.

g. From the points, as *e*, draw lines parallel to *fg* till they intersect the last named vertical lines.

h. Make $ns = 5'-9''$. Make the short vertical distance at $c = 4''$ draw sc, and make the upper side of the strut parallel to sc, and 4" from it. Note the intersection of this parallel with the line to the left of D, and connect this point with the upper end of c, to complete the strut.

i. Draw the edges of the rafter, parallel to those of the principal, 4" apart, and leaving 4" between the rafter and the principal. At *o*, draw a vertical line till it meets the lower edge of C, and from this intersection draw a horizontal line till it meets the upper edge of C; which gives proper dimensions to the wall plate.

j. From the intersections of the upper edges of the rafters, lay off downwards on the centre line 12", and make the ridge pole, thus located, 3" wide.

k. In the middle of the upper edge of the principal, place the purlin $4'' \times 6''$, and setting 2" into the principal.

l. Let the strap, *ab*, be 2" wide, and 2'—6" long from the bottom of the tie beam. Let it be spiked to the king post and tie beam, and let it be half an inch thick, as shown below the beam. W, the supporting wall, is made at pleasure.

Execution.—This mainly explains itself. As working drawings usually have the dimensions figured upon them, let the dimensions be recorded in small hair line figures, between arrow heads which denote what points the measurements refer to.

246. Ex. 4°. **A "Queen Post Truss" Bridge.** Pl. XXI., Fig. 126.

Mechanical construction.—This is a bridge of 33 feet span, over a canal 20'—6" wide between its banks at top, and 20'—2" at the water line. It rests on stone abutments, R and P, one of which is represented as resting on a plank and timber foundation, the other on "piles."

A is the tie beam; B, B' the *queen posts;* C, C' the principals; D the *collar beam*, or *straining sill;* R, P, the abutments; *eQt* the pavement of the *tow path;* *t*K the stone side walls of the canal; TT the opposite timber wall, held by timbers UU', N, dovetailed into the wall timbers; E, S, the piles, iron shod at bottom. These are the principal parts.

247. *Graphical construction.*—Let the scale be one of five feet to the inch.

a. All parts of the truss are laid off on, or from, the centre line AD. A is 14" deep; the dimensions of BB' are $12'' \times 6'$, except at top, where they are 10" wide for a vertical space of 16". C and D

are each 10″ deep. BB′ are 10′ apart, and the feet of C and C′, 12′ from the ends of the tie beam, which is 36′ long. D is 6″ below the top of the queen posts. *rr* are inch rods with five inch washers, $\frac{3}{4}$″ thick, and nuts $2\frac{1}{2}'' \times 1''$. *bb′* is a $\frac{3}{4}$″ bolt; with washer $4'' \times \frac{3}{4}''$ and nuts, $2'' \times 1''$; and perpendicular to the joint, *ad*.

b. From each end of the tie beam, lay off 1′—9″ each way for the width of the abutments, at the top. Make the right hand abutment rectangular in section and 11′ high, of rectangular stones in irregular bond (76). Let the left hand abutment have a batter of 1″ in 1′ on the side towards the canal, and let it be eleven feet high, in eleven equal courses.

c. Make *et*, the width of the paved tow path = 7′—6″, with a rise in the centre, at Q, of 6″.

d. The side wall is of rubble, 4′ thick at bottom, and extending 18″ below the water, with a batter of 1″ in 1′, and having its upper edge formed of a timber 12″ square.

e. The right hand abutment rests on a double course of three-inch planks, *qq′*, $5\frac{1}{2}$′ broad, and resting on four rows of 10″ piles, ES. S is the sheet iron conical shoe at the lower end of one of these piles, the dots at the upper end of which represent nails which fasten it to the pile.

f. TT is a timber wall having a batter of 1″ to 1′, and held in place by timbers, UU′, N, dovetailed into it at its horizontal joints, in various places.

g. The water line is 2′ below T*t*, and the water is $4\frac{1}{2}$ feet deep.

248. *Execution.*—It is intended that this plate should be tinted, though, on account of the difficulty of procuring adequate engraved fac-similes of tinted hand-made drawings, it is here shown only as a finished line drawing, and as such, explains itself, after observing that as the left hand abutment is shown in elevation, it is dotted below the ground; while, as the right hand abutment is shown in section, it is made wholly in full lines, and earth is shown only at each side of it.

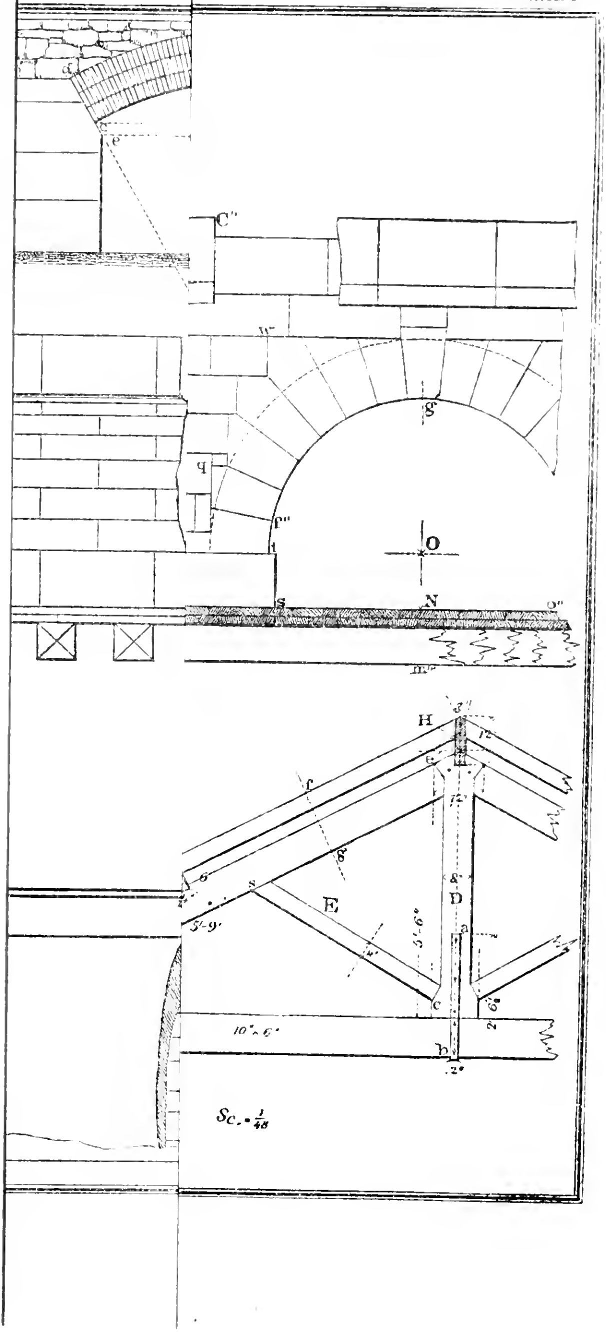
The usual conventional rule is, to fill the sectional elevation of a stone wall with wavy lines; but where other marks serve to distinguish elevations from sections, as in the case just described, this labor is unnecessary.

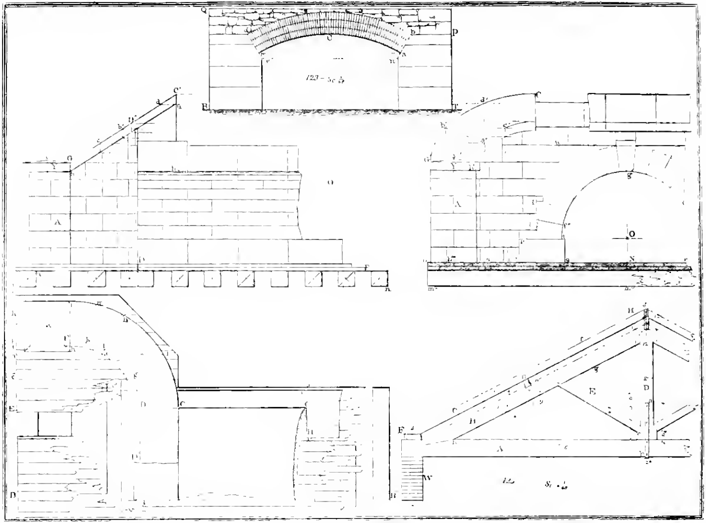
The following would be the general order of operations, in case this drawing were shaded.

a. Pencil all parts in fine faint lines.

b. Ink all parts in fine lines.

c. Grain the wood work with a very fine pen and light indian ink,


d
e
C''
w''
g
q
f''
t
O
N
s
o''
m''
H
e
f
g
6
s
E
D
a
c
b
5'-9'
5'-6''
10'' × 6'
Sc. = 1/48



the sides of timbers as seen on a newly-planed board, the ends of large timbers in rings and radial cracks, and the ends of planks in diagonal straight lines. See also the figures at *y*, where the lines of graining outside of the knots, are to extend throughout the tie beam.

d. Tint the wood work—the sides with pale clear burnt sienna, the ends with a darker tint of burnt sienna and indian ink.

e. Tint the abutments, and other stone work, with prussian blue mixed with a little carmine and indian ink, put on in a very light tint.

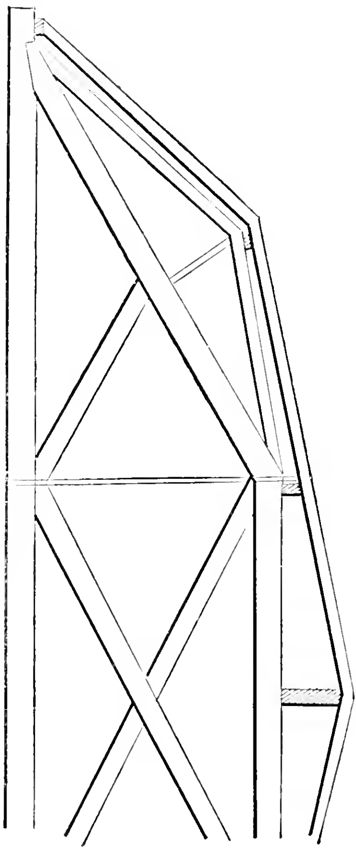
f. Grain the abutments in waving rows of fine, pale, vertical lines of uniform thickness, about one sixteenth of an inch long, leaving the upper and left-hand edges of the stones blank, to represent the mortar. The part of the left-hand abutment which is under ground is dotted only, as in the plate.

g. Grain the canal walls and paving, as shown in the plate, to indicate boulder rubble.

h. Shade the piles roughly, they being roughly cylindrical; tint them with pale burnt sienna, and the shoe, S, with prussian blue, the conventional tint for iron.

i. Rule the water in blue lines, distributed as in the figure.

j. Tint the dirt in fine horizontal strokes of any dingy mixture,

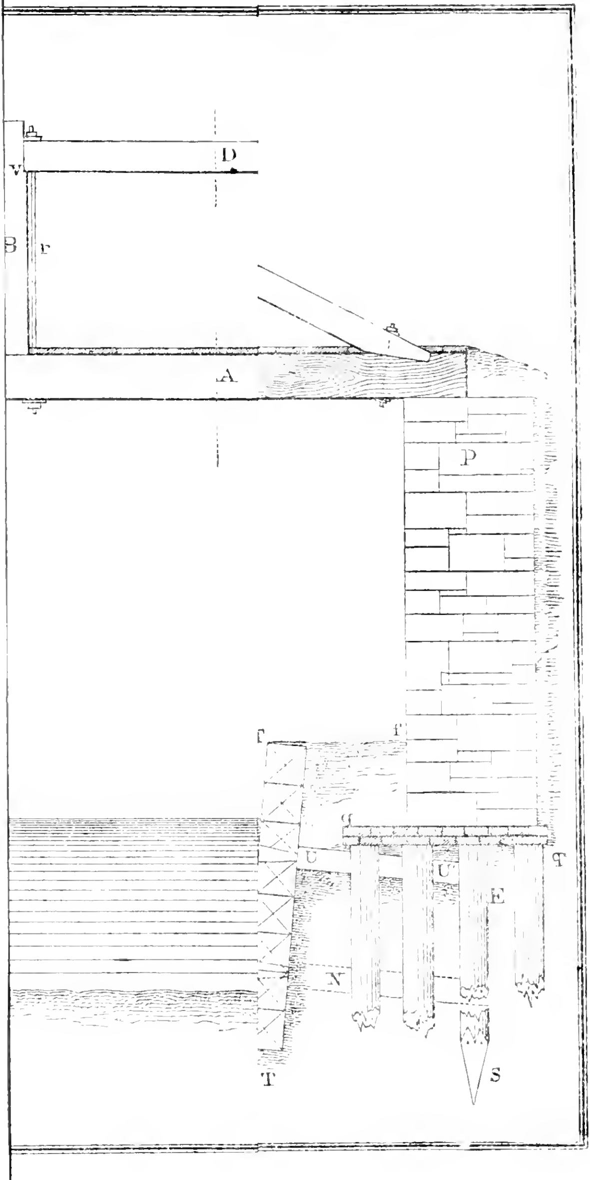


Note.—The above figure shows a little more than half of a queen-post *roof*-truss of 43 feet span. Omitting the light upper pieces, it may serve in place of Fig. 126 as a longer bridge truss; and may be drawn on any convenient scale from *four* to *six* feet to an inch.

in which burnt sienna prevails, in the parts above the water, and ink, in the muddy parts below the water, and then add, or not, the pen strokes shown in the plate, to represent sand, gravel, &c.

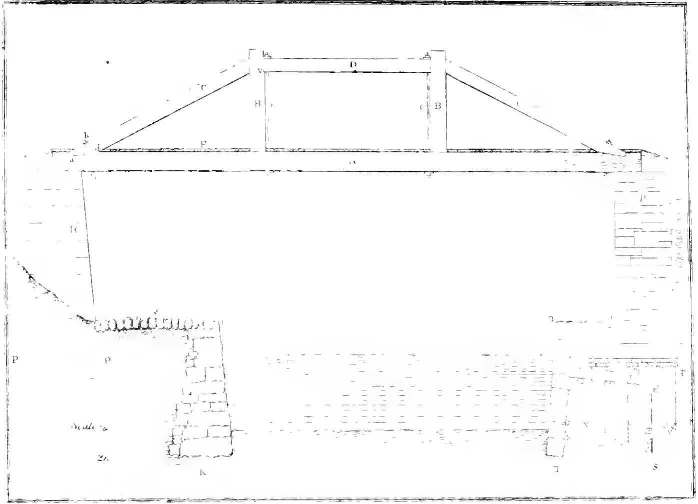
k. Place heavy lines on the right-hand and lower edges of all surfaces, except where such lines form dividing lines between two surfaces in the same plane. A heavy line on the under side of the floor planks, indicates that those planks project beyond the tie beam A.

X
PL. XXI.



are smooth and sunk into the plates so that their upper surfaces are

PL. XXI.



CHAPTER III.

IRON CONSTRUCTIONS.

252. Ex. 6°. A Railway Track. Pl. XXII., Figs. 129–134.

Mechanical construction, &c.—It may be thought an oversight to style this plate the drawing of a railroad track; but taking the track alone, or separate from its various special supports, as bridges, &c., its graphical representation is mainly summed up in that of two parts; *first*, the union of two rails at their joints; *second*, the intersection of two rails at the crossing of tracks, or at turn-outs. The fixture shown in Fig. 129, placed at the intersection of two rails to allow the unobstructed passage of car wheels, in either direction on either rail, is called a "Frog." Let y and z be fragments of two rails of the same track, then the side Hf' of the point of the frog, and the portion $k\,k'$ of its side flange, B, are in a line with the edges, denoted by dots, of the rails y and z, so that as the wheel passes either way, its flange rolls through the groove, I, without obstruction. When the wheel passes from y towards z there is a possibility of the flange's being caught in the groove, J, by dodging the point, f. To guard against this, a guard rail, $g\,g$, is placed near to the inside of the other rail, supposed to be on the side of the frog towards Fig. 132, as shown in the small sketch, Fig. 132, which prevents the pair of wheels, or the car-truck, from working so far towards the flange, B, as to allow the flange of the wheel to run into the groove, J, and so run off the track. $F\,f$, and the portion, $l\,l'$, of the flange, A, are in a line with the inner edge of the rail of a turn-out, for instance, the opposite rail being on the side of the frog towards the upper border of the plate, as shown in Fig. 132. Hence the flange of a car wheel in passing in either direction on the turn-out, passes through the groove, J, and is prevented from running into the groove, I, by a guard rail, near the inner edge of the opposite turn-out rail, as at U, Fig. 132.

253. Fig. 130 represents the under side of the right hand portion of the frog, and shows the nuts which secure one of the bolts which secure the steel plates, as D, E; bolts whose heads, as at u and v, are smooth and sunk into the plates so that their upper surfaces are

flush. It will be seen that there are two nuts on each bolt, as at D', on the bolt u—D D', which appears below the elevation, since it occurs between two of the cross-ties (sleepers) of the track. The nuts, as L, belonging to the bolt, b'', which are in the chairs, $q'p'$, w', x', are sunk in cylindrical recesses in the bottom of the frog, so as not to interfere with the cross-tie on which the surface, L, rests. The extra nut is called a check or "jam" nut. When screwed on snugly it wedges the first nut and itself also against the threads of the screw, so that the violent tremulous motion to which the frog is subjected during the rapid passage of heavy trains cannot start either of them.

In the end elevation, Fig. 131, A is the recess in the chair $x\ x'$, fitted for the reception of the rail, and B is the end of a rail in its place, as shown at y in the plan.

254. *Graphical Construction.*—From the above description it follows that the whole length of the frog depends on the shape of the part H f F, and the distance between this part and the side rails, as $c\ l$. In the present example $a\ c = 1'—11''$ and $c\ f = 20''$. $e\ d$ is $11''$ and $n\ k$ is $2''$ from F f. Having these relations given, and knowing that the lines at the extreme ends are perpendicular to the rails at those ends, the several figures of the frog can be constructed from the given measurements, without further explanation.

255. The construction of railway-track joints so as to secure as nearly as possible the uniform firmness of a continuous rail, has long exercised the minds of railway inventors. Cast-iron chairs, wrought-iron chairs, long chairs resting on ties each side of the joint, compound rails (Div. II., 140) solid-headed, or split through their entire height, and fish-joints have all been used; several of them in various forms. Fig. 133 is an isometrical drawing—scale $\frac{1}{12}$—of a wooden fish-joint which allows great smoothness of motion and freedom from the loud clack which accompanies the use of ordinary chairs. A, A, A, are the sleepers (cross-ties), D is a stout oak plank, perhaps six feet long, resting on three sleepers, and fitted to the curved side of the rail, as shown at d. This plank is on the outside of the track. On the inner side the rails are spiked in the usual way with hook-headed spikes $s\ s\ s$, of which those at the joint, r, pass through a flat wrought-iron plate, P, which gives a better bearing to the end of the rail, and prevents dislocation of parts. Each plank, as D, is bolted to the rail by four horizontal half-inch bolts, b, b, b, b, furnished with nuts and washers on the further side of D (not seen).

A modification of the above construction consists in substituting for the plate P, a short piece or strap of iron fitted to the surface of the inside of the rail, and through which the two bolts *bb*, next to the joint, pass.

With the now extended use of steel rails, the fish-joint, also in very general use, consists of an iron fish-plate on each side of the rail, with two bolts on each side of the joint. This makes a very firm joint. The plan has also been sometimes adopted of having the track break joints. That is, a joint, as *a*, Fig. 132, on one rail of a track, is placed opposite the centre of the rail *bc* of the other line of the same track. As a track always tends to settle at the joints, a jumping motion is induced in a passing train, which perhaps may be thought to be less violent if only on one rail at a time.

256. *Graphical Construction.*—Three lines through X, making angles of 60° with each other, will be the isometric axes. Remembering that it is the *relative* position of the lines which distinguishes an isometrical drawing, we can place XX′ parallel to the lower border, and thus fill out the plate to better advantage. The rail being 4″ wide at bottom, and 4″ high, circumscribe it by a square X*can*, from the sides of which, or from its vertical centre line, lay off, on isometric lines, the distances to the various points on the rail. Thus, let the widest part of the rail, near the top, be 3″ across, and ½ an inch below the top *ac*. Let the width at the top be 2″, and at the narrowest part 1″; and let the mean thickness of the lower flange be ¾″. The sides of the rail are represented by the bottom lines at XX′, and the tangents each side of R, to the curves of the section. Let the plank D be 6″ wide, and 4″ high. All the lines of the spikes, *ss*, are isometrical lines except their top edges, as *st*. The curve at the joint *r*, and at X′, are similar to the corresponding parts of the section at X.

To secure ease of graphical construction, let the bolt heads, *b*, &c., be placed so that their edges shall be isometric lines.

Fig. 134, is a plan and end elevation of a heavy cast-iron chair designed as a partial equivalent for a continuous rail, by making the outside of the chair extend to the top of the rail. The fault in every such contrivance, the best of which at present seems to be the fish-joint, is that, as the joint cannot be made as solid as the unbroken rail, the wave of depression just in advance of the engine is more or less completely broken at every joint.

258. Ex. 7°. **The Hydraulic Ram.** In order to give an iron construction, from the department of machinery, so as to render this volume a more fit elementary course for the machinist as well

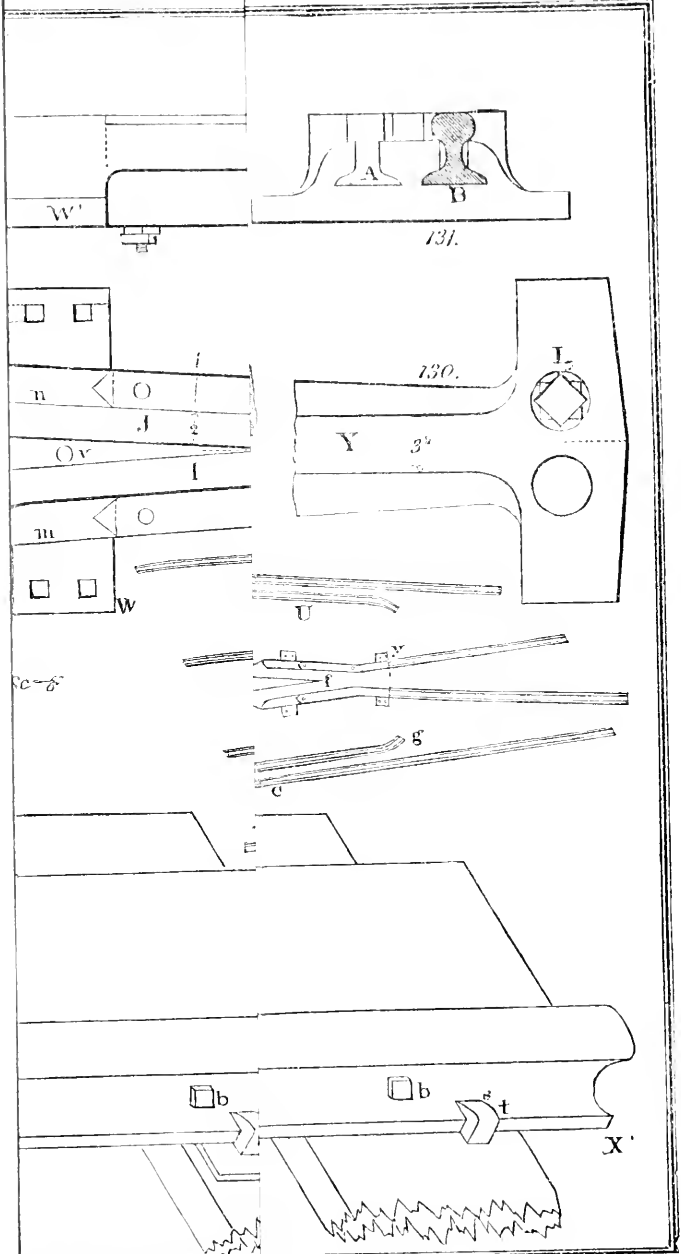
as for the civil engineer, a simple and generally useful structure viz. a hydraulic ram, has been chosen, as a fit example for the last to be described in detail.

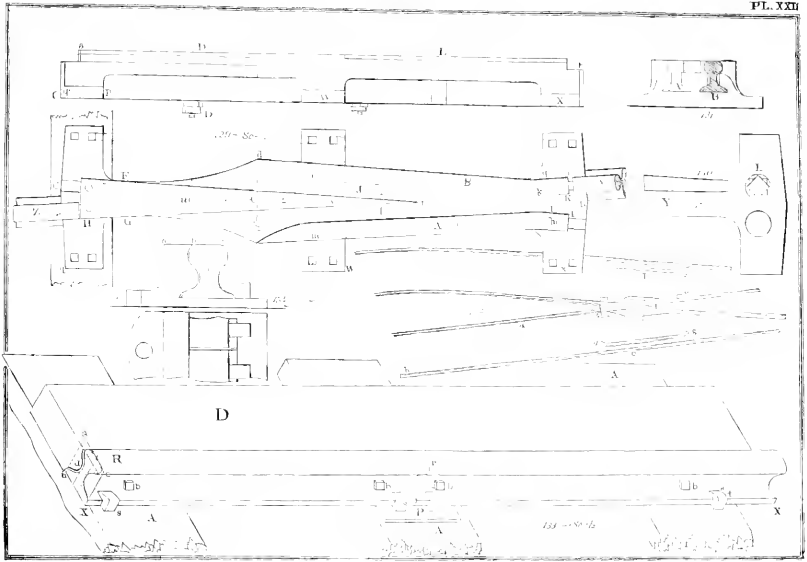
This machine is designed to employ the power of running water to elevate water to any desired height.

Pl. XXIII., Figs. 135–137, shows a hydraulic ram, of highly approved construction, and of half the full size.

259. *Mechanical construction.*—FF—F′F′ are feet to support the machine. These are screwed to a floor or other firm support. AB—A′B′B′ is the inlet pipe, opening into the air chamber C, at *a*—*a′b′* and ending at *dd*—*d′d′*—*d″d″* the opening in the top of the waste valve chamber, E—E′—E″. At *a*—*a′b′* is the opening as just noticed from the inlet pipe into the air chamber C (not seen in the plan). This opening is controlled by a leather valve *ee′*, weighted with a bit of copper *e″e‴*, and is fastened by a screw *h″h‴*, and an oblong washer *g′g*. At N and H are the extremities of two outlet pipes leading from the air chamber at F″F‴. Either one, but not both of these outlet pipes together, may be used, as one of the exchangeable flanges, H′ is solid, while the other is perforated, as seen at M′, Fig. 137. The air chamber is secured by bolts passing through its flange *f′f′*, through the pasteboard or leather packing, *pp*—*p′*, and the flange D—D′D′ at *cc*. This flange, and part of the inlet pipe are shown as broken in the elevation, so as to expose the valve *ee′*, and the adjacent parts. LL′ is a flange through which the inlet pipe passes, and this pipe is slit and bent over the inner edge of the aperture in LL′, forming a flange, which presses against a leather packing, *tt′*, and makes a tight joint. The outlet pipes are secured in the same way. At *uu*—*u′* are the square heads of bolts which fasten the flanges to the projections UU—U′. K—K′ is a shelf bearing the waste valve chamber, E—E′E″, and the adjacent parts. W—W′ is the flange of this valve chamber, secured by two bolts at *vv″*—*v′*, which pass through the leather packing *y*. *h′h″* is the waste valve, perforated with holes, *x*, to allow water to flow through it. *mm′* is the valve stem. *d′d′kk′* is a perforated standard serving as a guide to the valve stem, and also as a support to the hollow screw *s*. *n* is a rest, secured to the valve stem by a pin *p″*. *q″* is a nut, part of which, *qq′*, is made hexagonal. *r* is a "jam" nut (253).

In the plan of this portion of the machine, the innermost circle is the top of the valve stem; next is the body of the valve stem; next, the top of the rest; next, the bottom of the same; next, the


W'
A
B
131.
130.
L
n
J
l
m
W
Y
U
f
g
c
b
b
t
X'



nut q''; and outside of that, and resting on the top of the waste valve chamber, are the standards, *dd*.

260. *Operation.—Principles involved.*—In the case of what might be called *passive constructions*, that is mere stationary supports, like bridges, &c., a knowledge of the construction of the parts enables one to proceed intelligently in making a drawing; but, in the case of what may, in opposition to the foregoing, be called *active constructions*, or machines, a knowledge of their mode of operation is usually essential to the most expeditious and accurate graphical construction of them, because a machine consists of a train of connected pieces, so that a given position of any piece implies a corresponding position for every other part. Having, then, in a drawing, assumed a definite position for some important part, the remaining parts must be *located* from a knowledge of the machine, though *drawn* by measurements of the dimensions of that part. Only *fixed bearings*, and *centres of motion*, can properly be located by measurement, in machine drawing.

The principles involved in the operation of the hydraulic ram may be summed up under three heads, as follows:

261. I. *Work.* *a.* When a certain *weight* is moved through a certain *space*, a certain amount of *work* is expended.

b. Thus; when a quantity of water descends through a certain space, a certain amount of work is developed.

c. As the idea of work involves the idea both of weight moved, and space traversed, it follows that *works* may be equal, while the weights and spaces may be unequal. Thus the work developed by a certain quantity of water, while descending through a certain height, may be equal to that expended in raising a portion of that water to a greater height.

262. II. *Equilibrium.* *a.* Where forces are balanced, or mutually neutralized, they are said to be in equilibrium. Now the usual fact is, that when such equilibrium is disturbed, it does not restore itself at once, but gradually, by a series of alternations about the state of equilibrium. Thus a stationary pendulum, being swung from its position of equilibrium, does not, at the first returning vibration, stop at the lowest point, but does so only after many vibrations.

b. Theoretically, these vibrations, as in the case of the pendulum, would never stop, but in practice the resistance of the air, friction, &c., make a continual supply of a greater or less amount of force necessary to perpetuate the alternations about the position or state of equilibrium.

263. III. A *physical fact* taken account of in the hydraulic ram, is, that water in contact with compressed air will absorb a certain portion of such air.

264. Passing now more particularly to a description of the operation of the hydraulic ram: 1°. Water from some elevated pond or reservoir flows into the machine, through the inlet pipe AA' and continues through the machine, and flows out through the hole in the waste valve $h'h''$, pressing meanwhile against the solid parts of the roof of this valve, whose hollow form — open at the bottom — is clearly shown in Fig. 136.

2°. Presently the water acquires such a velocity as to press so strongly against the roof of the waste valve, that this valve is lifted against the under side of the roof of its chamber which it fits accurately.

3°. The water thus instantly checked, expends its acquired force in rushing through the valve e—$e'e'''$ and in compressing the air in the air chamber C.

4°. The holes F'' or F''' of the outlet pipe, leading to an unobstructed outlet, the compressed air immediately forces the water out through the outlet pipe until, after a number of repetitions of this chain of operations, the portion of the water thus expelled from the air chamber is raised to a considerable height.

5°. In accordance with the second principle, the flow of water from the air chamber does not cease at the moment when the confined air is restored to its natural density, but continues, so that—taking account also of the absorption of the air by the water at the time of compression—for a moment the air of the air chamber is more rare than the external atmosphere. Hence to keep a constant supply of air to the air chamber, a fine hole called a snifting hole, is punctured, as with a needle, at ss', i.e., just at the entrance of the inlet pipe into the machine. Through this hole air enters, with a snifting sound, when the flow of water recommences, so as to supply the air chamber with a constant quantity of air. When the waste valve is at the bottom of the chamber EE', the nut and "jam" are together at the bottom of the screw s', and the valve is at liberty to make a full stroke. By raising the valve to its highest point and turning the nut and "jam" to some position as shown in the figure, the stroke of the valve can be shortened at pleasure, and, at its lowest point, will be as far from the bottom of the chamber as the "jam," q'', is above its lowest position.

266. In practice, it is found that the strokes of the waste valve shortly become regular; their frequency depending in any given

case on the height of the supply reservoir, the height of the ejected column, the size of the machine, the length of the stroke of the valve, &c.

267. The proportion of water discharged into the receiving eservoir will also depend on the above named circumstances, being more or less than one third of the quantity entering the machine at AA′. In a machine by M. Montgolfier of France, said to be the original inventor—water falling $7\frac{1}{2}$ feet, raised $\frac{2}{21}$ of itself to a height of 50 feet.

268. *Graphical Construction*.—Scale; half the full size. *a.* Having the extreme dimensions of the plan, in round numbers 9″ and 12″, proceed to arrange the ground line, leaving room for the plan below it.

b. Draw a centre line, NC, for plan and elevation, about in the middle of the width of the plate.

c. Draw a centre line, AK, for the plan, parallel to the ground line.

d. Exactly $4\frac{1}{2}$″ from the centre line NC, draw the centre line *vv″*—K′*m′* for the waste valve chamber and parts adjacent.

e. With the intersection, *, of the centre lines of the plan, as a centre, draw circles having radii of $1\frac{3}{4}$″ and $3\frac{1}{16}$″ respectively, and through the same centre, draw diagonals, as *cc*.

f. On the centre line, NC, are the centres of the circles, F″F‴, whose circumferences come within $\frac{1}{10}$ of an inch of the inner one of the two circles just drawn.

g. Draw the valve, *e*, the copper weight *e″*, the screw end, *h*, and the nut and oblong washer, *h″* and *g*.

h. Locate, at once, the centres of all the small circles, *cc*, &c., by the intersection of arcs, $\frac{1}{4}$″ from the circle *pp* having * for a centre, with the diagonals; then proceed to draw these circles.

i. Draw the projections, as U, drawing the opposite ones simultaneously, and using an auxiliary end view of the nuts *u*, as often explained before.

j. Draw the feet, F, with their grooves, F, and bevel edged screw holes, L″.

k. In drawing the shelf, K, and flange W, the intersection of the centre lines BK and *m′m*, is the centre for the curves which intersect the centre line AK; the corners, 1 1, of the nuts, *v*,*v″*, are the centres for the curves that cross the centre line, *vv″*; and the remaining outlines of the shelf are tangents to the arcs thus drawn, and those of the flange are lines sketched in so as to give curves tangent to the arcs already drawn, and short straight lines parallel to *vv″*

l. The remaining circles and larger hexagon, u', of this portion of the plan, have the intersection of the centre lines for a centre; and may be drawn by measurements independently of the elevation, or by projection from the elevation, after that shall have been finished.

269. Passing to the elevation:—

a. Construct, at one position of the T square, the horizontal lines of both feet; then the horizontal lines of the nuts u', and flange L′, and projection U′; with the horizontal lines of the floor of the air chamber and adjacent parts.

b. Project up from the plan the vertical edges of the feet, F′F′, the flange, nut, and projection L′, u' and U′, the valve e', the copper e'', the screw h'', the washer g, the air chamber flange $f'f''$, and screw z. Break away the portion D—see plan—of the body of the machine, and the near wall of the water channel A′B′. Break away also the further wall of the water channel so as to show a section, II′, of the further outlet pipe, II—see plan. Q is the centre of the spherical part of the air chamber to which the conical part is tangent.

c. Draw all the horizontal lines of the waste valve chamber and parts adjacent. Make the edges of the threads of the screw straight and slightly inclined upwards toward the right.

d. Project up from the plan, or lay off, by measurement, the widths of various parts through which the valve stem passes, and draw their vertical edges.

Fig. 136 is a section of the waste valve chamber, showing part both of the interior and exterior of the waste valve. The dotted circles form an auxiliary plan of this valve, in which the holes have two radial sides, and two circular sides with x'' as a centre. The top of the valve is conical, so that in the detail below, two of the sides of the hole n, tend towards the vertex, x. At n', one of these holes, of which there are supposed to be five, is shown in section.

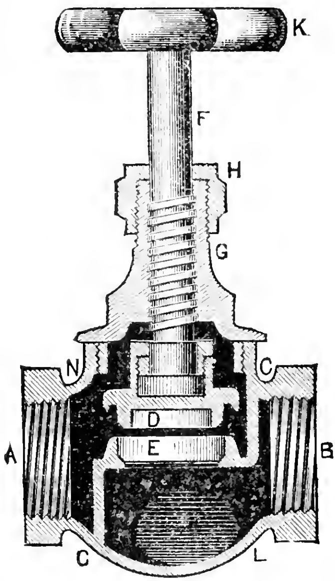
Fig. 137. The outlines of M, one of the outlet pipe flanges, are drawn by processes similar to those employed in drawing the shelf, K, in plan.

270. *Execution.* As a line drawing, the plate explains itself. It would make a very beautiful shaded drawing and one that the careful student of the chapter on shading and shadows, would be able to execute with substantial accuracy, without further instruction.

We conclude this division with the following additional exercises as examples of iron constructions—one from civil engineer-

ing practice, the other three from mechanical engineering; styling them exercises, since, being partially shown (yet, with the description, sufficiently so for their purpose), they leave something to be supplied by the student from the general insight gained from previous practice.

Exercise 1. *A Stop-valve.*—The following figure shows one of many forms of valve differing more or less in detail, and made for the purpose of shutting off the passage of steam, water, etc., through pipes. Such



valves either *lift* from their seats, as in the example shown, or *slide* off them, in which case they are sometimes called *gates*.

The figure represents what is called a *globe-valve*, from the general external form of its *valve-chamber* NCCL. In this chamber is a bent partition, or diaphram, CEC, containing the seat, E, of the valve D. This valve is raised or lowered by means of the hand-wheel K, and screw valve-stem F working in the collar G, which is screwed into the top, NC, of the chamber.

The head at the bottom of the valve-stem, working loosely in the hollow head of the valve, raises the latter vertically without turning it. The cap H secures the necessary packing. Opening the valve then allows of the passage of any fluid through AB and the pipes which may

be attached at A and B. These openings are from $\frac{1}{4}''$ to $2''$ diameter. The measurements and scale may therefore be assumed, and plan and end elevation added.

Exercise 2. *An iron truss bridge.* Pl XXIV., Figs. 1–7.—This bridge is partly of wrought, and partly of cast iron, and known from its form and its inventor as *Whipple's trapezoidal-truss bridge.*

The *upper chords*, a—$a'a'$, are hollow cast-iron cylinders $7\frac{1}{2}''$ diameter, and $\frac{1}{2}''$ to $\frac{5}{8}''$ thickness of metal. The *posts*, p', and *struts*, S, S′S″, are also of cast-iron, the latter, double, as seen in the fragment of end elevation, Fig. 2, and fragment of plan, Fig. 3.

The posts extend through the flooring, where they are $5''$ in diameter, and rest on seats on the tops of the cast-iron *coupling blocks*, $n'n'$, as shown in the plan, Fig. 5, and end elevation, Fig. 6, of one of these blocks.

The *lower chord*, bb—$b'b'$, is composed of heavy wrought-iron rods made in links embracing two successive coupling-blocks, in the manner shown in Figs. 4–6. The two end lengths, however, are single, as shown, and are secured by nuts, g', at the outer end of the shoes s, s'', which holds the feet of the struts SS′S″.

The structure is further held in shape, and the forces acting in it suitably sustained and distributed by the diagonal and vertical rods $r'r'$, each of which, after the first two from the end, crosses two *panels* of the bridge, as the spaces between the posts are called.

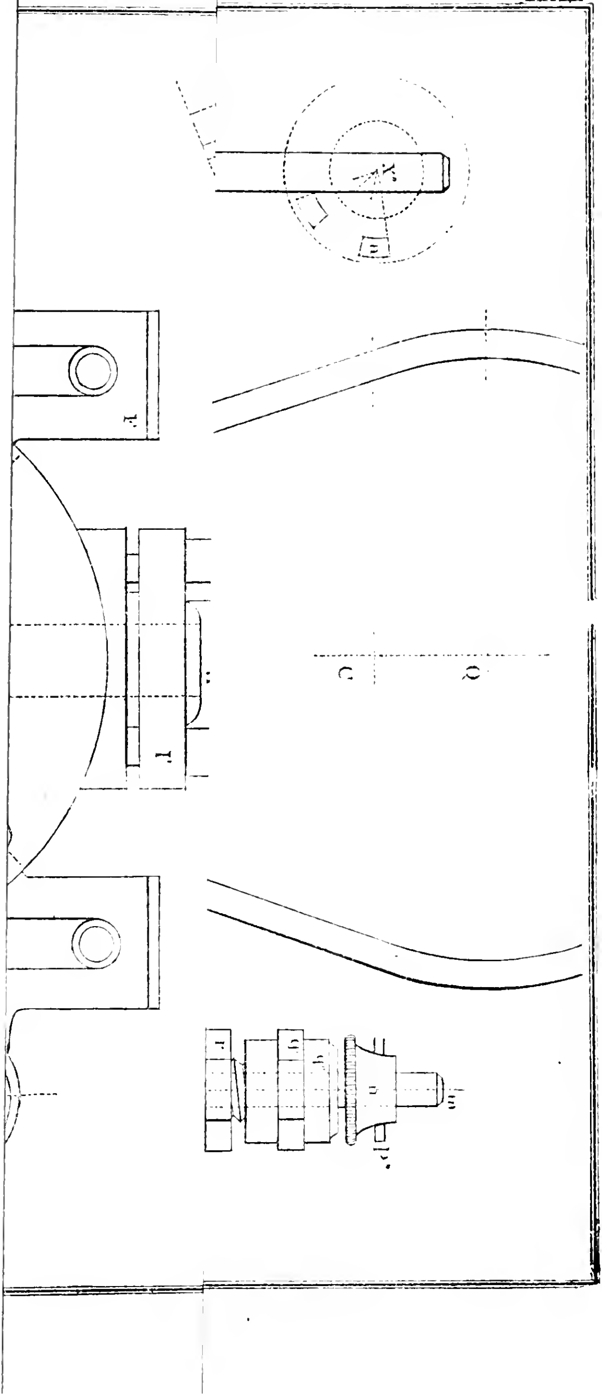
The horizontal diagonal rods, f, under the floor, tightened by links l working on right- and left-handed screws (Div. V., Ex. 9) in the adjacent rod ends, provide against the horizontal force of winds. The light transverse flanged beams k—k'', overhead, also help to stiffen the structure laterally.

The main transverse beams c—c'—c'' rest on the coupling-blocks, and support the floor joists $dd'd''$, on which the floor planks $gg'g''$ rest. CC′C″ is the coping, serving to cover the irregular ends of the floor planks, and as a guard to prevent vehicles from striking the truss.

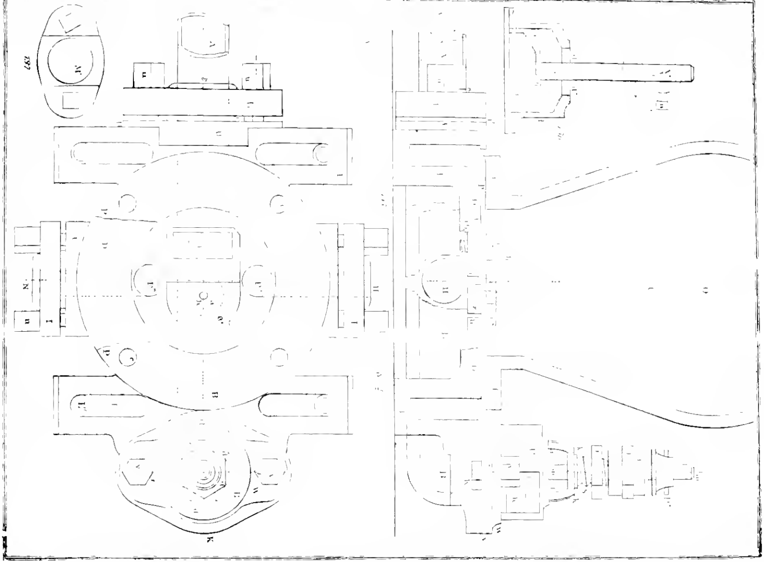
Fig. 7 is an enlarged view of the centre joint where the two halves of the posts meet.

Other useful details would be vertical longitudinal sections of the joints as ee'' and e', which would show an opening in the under side of the upper chord, sufficient for the entrance of the diagonal rods, and these rods forged into rings clasping the stout wrought-iron pins e, $e'e''$; also the level bearing for the head of the post, except at the joint at the head of the strut.

The three top cross-beams indicated at kh, Fig. 3, show that a *skew-bridge* is represented, that is, one which crosses the stream obliquely, the extreme timber, h, being parallel to the length of the stream.



PL. XXIII



The span of the bridge is 114 feet, in 12 equal panels of 9½ feet each; the roadway is 19 feet wide from centre to centre of the trusses, which are 15 feet 9 inches in height from the centre of the coupling-blocks, n', to that of the upper-chord pins as at c'.

Suitable scales are 3 to 5 feet to 1 inch for the general views, and from 6 inches to 1 foot to one inch for the details.

Exercise 3. *A vertical boiler.* Pl. XXIV., Fig. 8.—This figure, being given partly as an excellent example in shading, and of certain flame effects instructive to the draftsman, is described without letters of reference.

The figure represents a vertical section of what is known as the Shapley patent boiler, differing from the ordinary tubular vertical boiler as appears from the figure and following description.

The central combustion chamber, being tall, is designed to effect three results; viz., to raise its top, called the *crown sheet*, so far above the fire as to retard burning out; to afford abundant room for perfect combustion, thereby generating more heat; and to effectually convey this heat to the water which surrounds the fire-box in a thin sheet.

Heat is further conveyed to the water by passing, as shown by the arrows, through short transverse tubes, two of them seen in section, and vertical tubes between the fire-box and the outer shell. These open into the annular base flue (interrupted by the ash-pit door), which leads to the smoke-pipe (sometimes called the up-take).

The upper section, or steam-dome, is mostly occupied by steam, and is stayed by bolts to the crown sheet.

Since the tubes, when sooty, lose much of their heat-conducting power, they are, in this boiler, made very easily accessible for frequent cleaning by connecting the two sections of the boiler by a double annular jacket which contains no steam or water and sustains no pressure. It is made in sections for easy removal, and thus allows ready access to the tubes.

Exercise 4. *A direct-acting steam-pump.* Pl. XXIV., Fig. 9.—The magnitude and variety of pumping requirements for water, oil, and various other liquids, hot or cold, thin or viscid, pure or gritty, and for drainage, mining, city, hotel, railroad-station, and other purposes, have called forth a large amount of inventive talent and many ingenious and effective pumping engines.

The figure represents a vertical longitudinal section of the Knowles steam-pump, affording a useful study and guide in making a finished drawing.

BB is the pump barrel or water cylinder—lined, when the character of the fluid to be pumped requires it, with composition linings, shown at xx and similarly in section on the upper side of the barrel. P is the

water piston with its packing p, and secured to the piston-rod a by nut and lock-nut (Div. V., 12) seen at the left.

JEF is the steam cylinder with its piston on the same piston-rod, a, with the water piston, thus forming what is called a direct-acting pump. Both cylinders are provided with stuffing-boxes hg and K.

The pump valves under the letter b are here shown as lifting disk valves circular in plan, but may be cage, or hinge, or any other valves.

In the position shown, and the piston still moving towards the left, water is entering through the lower or *suction* circular inlet and the lower right-hand valve, and is discharging by the upper or discharge pipe, which is smaller than the suction pipe. By raising the upper left-hand valve the discharge water also partly enters the air-chamber A, where, by compressing the confined air, a steady discharge is obtained. The valves rise and fall, each working on a short spindle, and are quickly closed by the aid of spiral springs above them; seen on the two closed valves.

The steam and exhaust ports and passages to the steam cylinder are of the usual form ; n, the orifice for the admission of steam from the boiler, and the central orifice is the exhaust. The steam-valve is a double D valve.

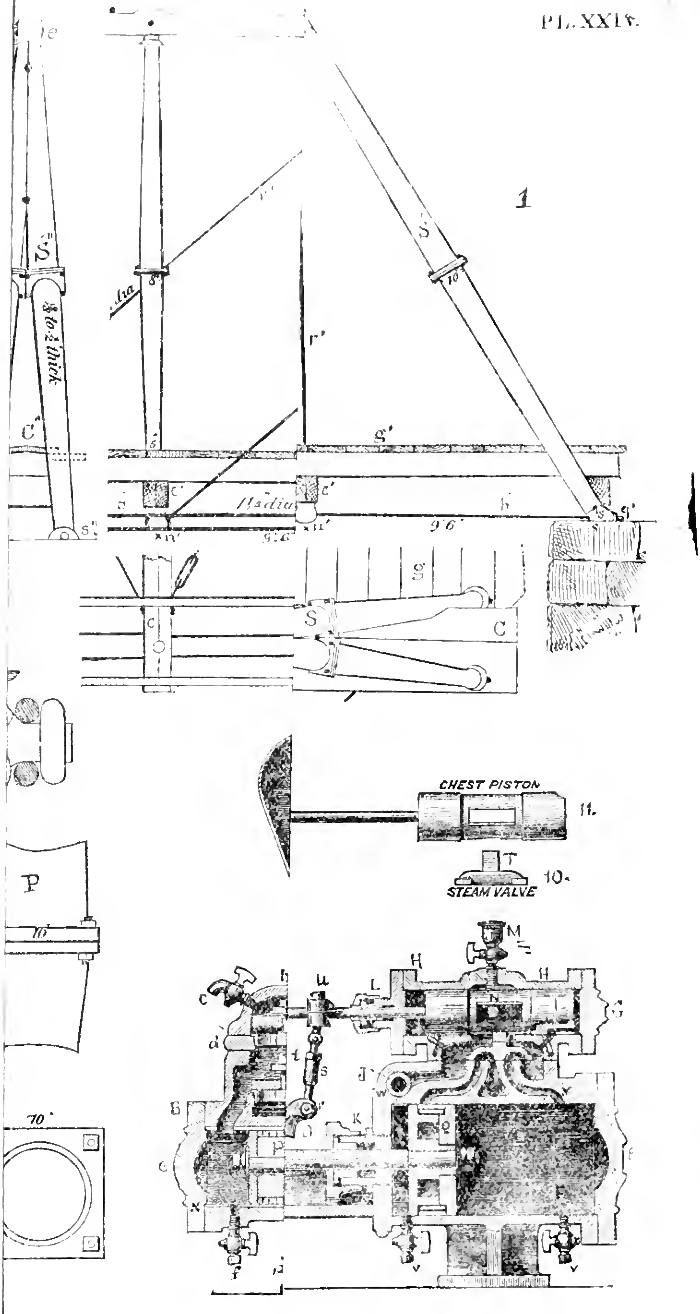
A stroke to the right being about to begin, a roller on the opposite side of the tappet arm CC, carried by the piston-rod a, raises the left end of the rocker DD. This, by means of the link s, slightly rotates the valve-rod l and its "chest-piston," Fig. 11, so as to bring it into a position to take steam through the small passage at the lower right-hand corner of the steam-chest G, which throws the piston to the opposite end of its stroke, carrying the valve by means of its stem T, Fig. 10. Steam can then enter the left-hand end of the cylinder through the left-hand chamber of the D valve, while exhaust steam escapes through the passage y and the right-hand chamber into the central, or exhaust passage.

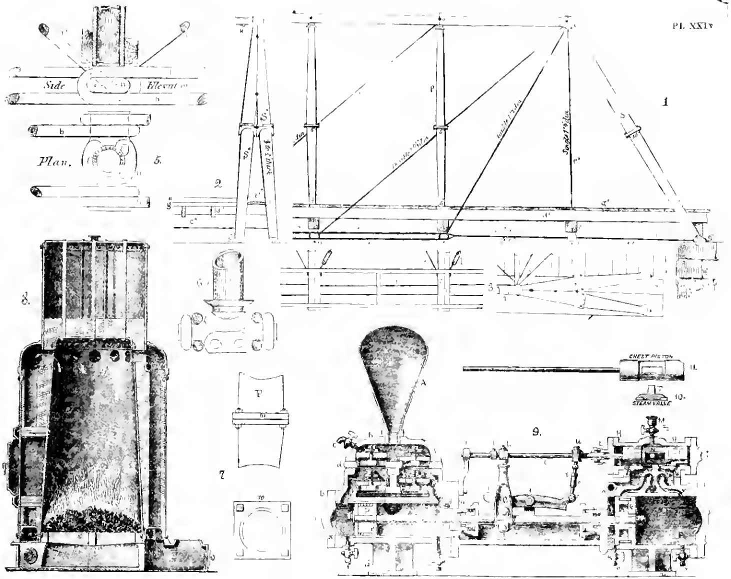
At i is the valve-rod guide. j is a collar on the valve-rod. u clamps the rocker connection to the valve-rod. t adjusts the link s. M is the oil-cup, and n a stud to attach a hand lever.

These pumps are made of a large range of sizes, from water cylinders of 2″, and steam cylinders of 3¼″ diameter, and 4″ stroke; to water cylinders of 20″, and steam cylinders of 28″ diameter, and 12″ stroke. Fig. 9 may be regarded for drawing purposes as a sketch (from a scale drawing and in true proportion, however) of a pump having a water cylinder of 7″, and a steam cylinder of 12″ diameter, with a 12″ stroke.

For variety of practice in the use of scales, the pump may then be drawn on a scale of $\frac{1}{8}$ or $\frac{1}{6}$ the full size, with details on scales of 3″ to a foot, or of full size.

THE END.


1
S'
10
8
r'
g'
c'
C
S
1 1/4 dia
9'6"
3/8 to 1/2 thick
P
CHEST PISTON
11.
T
10.
STEAM VALVE
M
H
H
G
L
u
h
c
d
t
s
J
w
B
K
E
v
v
f


Side
Plan.
CHEST PISTON
STEAM VALVE

John E. Sjoquist